

March 2, 2006

Name \_\_\_\_\_

The total number of points available is 138. Throughout this test, **show your work.**

1. (12 points) Let  $f(x) = \sqrt{x^3 - x + 3}$ .

(a) Compute  $f'(x)$

(b) What is  $f'(2)$ ?

(c) Use the information in (b) to find an equation for the line tangent to the graph of  $f$  at the point  $(2, f(2))$ .

2. (12 points) Consider the function  $f$  defined by:

$$f(x) = \begin{cases} 3x - x^3 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x^{2/3} & \text{if } x > 1 \end{cases}$$

(a) Is  $f$  continuous at  $x = 1$ ?

(b) What is the slope of the line tangent to the graph of  $f$  at the point  $(8, 8)$ ?

(c) Find  $f'(-3)$

3. (12 points) If a ball is thrown vertically upward from the roof of 112 foot building with a velocity of 48 ft/sec, its height after  $t$  seconds is  $s(t) = 112 + 48t - 16t^2$ .
- (a) What is the height the ball at time  $t = 0$ ?
- (b) What is the velocity of the ball at the time it reaches its maximum height?
- (c) What is the maximum height the ball reaches?
- (d) What is the velocity of the ball when it hits the ground (height 0)?
4. (7 points) The cost of producing  $x$  units of stuffed alligator toys is  $C(x) = 0.003x^2 + 6x + 6000$ . Find the marginal cost at the production level of 1000 units.

5. (30 points) Consider the table of values given for the functions  $f, f', g,$  and  $g'$ :

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let  $L(x) = f(x) \cdot g(x)$ . Compute  $L'(2)$ .

(b) Let  $U(x) = g \circ g(x)$ . Compute  $U'(1)$ .

(c) Let  $K(x) = g(x^2) \cdot f(x)$ . Compute  $K'(1)$

(d) Let  $V(x) = f(g(2x))$ . Compute  $V'(3)$ .

(e) Let  $W(x) = [g(2x - f(x))]^2$ . Compute  $W'(4)$ .

(f) Let  $Z(x) = g(x^2 + f(x))$ . Compute  $Z'(1)$ .

6. (25 points) Compute the following derivatives.

(a) Let  $f(x) = (x + \sqrt{1 + x^3})^2$ . Find  $\frac{d}{dx}f(x)$ .

(b) Let  $g(x) = x^3/(1 + x^2)$ . What is  $g'(x)$ ?

(c) Find  $\frac{d}{dx}((x + 2)^2 \cdot (2x - 1))$ .

(d) Find  $\frac{d}{dx}\sqrt{\frac{2x^3+1}{3x-2}}$ .

(e) Find  $\frac{d}{dt}(t^2 + 1/t^2)^4$ .

7. (40 points) Consider the rational function

$$r(x) = \frac{(x^2 - 4)(2x + 1)}{(3x^2 - 3)(x - 2)}.$$

Use the Test Interval Technique to solve the inequality  $r(x) \geq 0$ .