October 3, 2013 **Name** ______________________________

The problems count as marked. The total number of points available is 171. Throughout this test, for full credit you must **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (6 points) Find an equation in slope-intercept form for a line parallel to the line $3x - 6y = 7$ which goes through the point $(-3, 5)$.

2. (10 points) What is the smallest possible value of the expression

$$|x-1| + |x-2| + |x-4|?$$

3. (10 points) The set of points satisfying $(x-1)^2 + (y-2)^2 = 16$ is a circle. The set of points satisfying $x^2 + 4x + y^2 + 6y = 100$ is also a circle. What is the slope of the line connecting the centers of the two circles?

4. (35 points) Evaluate each of the limits (and function values) indicated below.

   (a) $\displaystyle\lim_{x\to 6} \frac{\sqrt{2x-3}-3}{x-6}$

   (b) $\displaystyle\lim_{x\to 2} \frac{3x-6}{\frac{1}{2x}-\frac{1}{4}}$

   (c) $\displaystyle\lim_{x\to 3} \frac{x^3-3x^2}{x^2-2x-3}$

   (d) $\displaystyle\lim_{x\to \infty} \frac{(2x-3)^3}{x(4x-1)^2}$

   (e) $\displaystyle\lim_{x\to 0} \frac{(x+1)^3-1}{x}$

5. (30 points)

The following ten problems are worth 3 points each. For problems (a) through (j), let

$$f(x) = \begin{cases} 2x+1 & \text{if } -3 \le x < -1 \\ 3x-1 & \text{if } -1 \le x \le 2 \\ x+3 & \text{if } 2 < x \le 4 \\ 1 & \text{if } 4 < x \le 6 \end{cases}$$

Find the value, if it exists, of each item below. Use DNE when the value does not exist.

(a) What is the domain of the function f. Express your answer in interval notation.

(b) $\lim_{x \to -1^-} f(x)$

(c) $\lim_{x \to -1^+} f(x)$

(d) $\lim_{x \to -1} f(x)$

(e) $f(-1)$

(f) $\lim_{x \to 2^-} f(x)$

(g) $\lim_{x \to 2^+} f(x)$

(h) $\lim_{x \to 2} f(x)$

(i) $f(2)$

(j) $\lim_{x \to 4} f(x)$

6. (15 points) Let $H(x) = (\sqrt{x^2 - 1} - 2)^3$.

   (a) What is the (implied) domain of H?

   (b) Find five functions, f, g, h, l, and k so that $H(x) = f \circ g \circ h \circ l \circ k(x)$.

   (c) Compute $H'(x)$.

7. (10 points) If $g(x) = (x^2 - 1)^2(2x + 1)^3$, then

$$g'(x) = 4x(x^2 - 1)(2x + 1)^3 + 6(x^2 - 1)^2(2x + 1)^2.$$

   Find all the x-intercepts of the function $g'(x)$.

8. (20 points) Let $f(x) = \sqrt{3x+1}$. Notice that $f(5) = \sqrt{3 \cdot 5 + 1} = 4$.

   (a) Find the slope of the line joining the two points $(4, f(4))$ and $(5, f(5))$.

   (b) Let h be a positive number. What is the slope of the line passing through the points $(5, f(5))$ and $(5+h, f(5+h))$. Your answer depends on h of course.

   (c) Compute $\lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$ to get $f'(5)$.

   (d) Your answer to (c) is the slope of the line tangent to the graph of f at the point $(5, f(5))$. In other words, your answer is $f'(5)$. Write and equation for the tangent line.

9. (20 points) Let $G(x) = \sqrt{(x-4)(2x+1)(x+3)(x+5)}$

   (a) Find the domain of G and express it as a union of intervals (in interval notation).

   (b) You might have used $x = 5$ as a test point in part a. On the other hand you might have used $x = 6$. Given that the function $F(x) = (x-4)(2x+1)(x+3)(x+5)$ is continuous over the real numbers, explain why the Intermediate Value Theorem guarantees that the sign of $F(5)$ is the same as the sign of $F(6)$.

10. (15 points) Find a (symbolic representation for a) quadratic polynomial whose graph includes the points $(-1, 0), (3, -16)$ and $(5, 0)$.