February 6, 2013 **Name** ______________________________

The problems count as marked. The total number of points available is 149. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (30 points) Let L denote the line defined by the equation $2y + 3x = 12$.

   (a) Select a point P that belongs to the line. There are infinitely many correct answers to this.

   (b) Find an equation for the line perpendicular to L that goes through the point you selected.

   (c) Find the distance between your point P and the origin $(0, 0)$.

   (d) Find the midpoint of the line segment with endpoints P and $(0, 0)$.

2. (35 points) Evaluate each of the limits indicated below.

(a) $\lim\limits_{x \to \infty} \dfrac{(2x^2 - 3)^2}{(x - 1)^4}$

(b) $\lim\limits_{x \to 4} \dfrac{x - 4}{x^2 - 16}$

(c) $\lim\limits_{h \to 0} \dfrac{(4 + h)^2 - 16}{h}$.

(d) $\lim\limits_{x \to 1} \dfrac{x^2 + 3x - 4}{x^2 - 3x + 2}$

(e) $\lim\limits_{x \to 2} \dfrac{\frac{1}{4x} - \frac{1}{8}}{\frac{1}{2x} - \frac{1}{4}}$

(f) $\lim\limits_{x \to 5} \dfrac{\sqrt{3x + 1} - 4}{x - 5}$

3. (12 points) Consider the function g defined symbolically by

$$g(x) = \sqrt{(x+3)(2x-1)(x-5)}.$$

Note that $g(0) = \sqrt{15}$, so 0 belongs to the domain of g. Find the domain of the function. Express your answer as a union of intervals. That is, use interval notation.

4. (12 points) Let $H(x) = (x^2-1)^2(x+2)^2$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2-1) \cdot 2x(x+2)^2 + 2(x+2)(x^2-1)^2.$$

Please note that the derivative has already been found for you. There is no need to differentiate. Find all of the zeros of $H'(x)$. This is not a calculus problem. It's an algebra problem.

5. (10 points) The demand curve for a new phone is given by $3p+2x=12$ where p is the price in hundreds of dollars and x is the number demanded in millions. The supply curve is given by $x-p^2+4p=4$. Find the point of equilibrium.

6. (10 points) Find the exact value of $|2\sqrt{2}-7|+|1-6\sqrt{2}|-|3\sqrt{2}-11|$.

7. (15 points) Let

$$f(x) = \begin{cases} |x-2| & \text{if } x < -1 \\ 2x+5 & \text{if } -1 \leq x \leq 1 \\ x-2 & \text{if } 1 < x \leq 3 \\ x^2 - 8 & \text{if } 3 < x \end{cases}$$

(a) What is $f(-1)$?

(b) What is $f(1)$?

(c) What is $f(3)$?

(d) What is $\lim_{x \to -1^-} f(x)$?

(e) What is $\lim_{x \to 1^+} f(x)$?

(f) What is $\lim_{x \to 3} f(x)$?

(g) List the x-values between -2 and 4 for which f is not continuous.

8. (25 points) Let $f(x) = \frac{1}{2x+1}$. Notice that $f(0) = 1$.

   (a) Find the slope of the line joining the two points $(0, f(0))$ and $(1, f(1))$.

   (b) Let h be a positive number. What is the slope of the line passing through the points $(0, f(0)$ and $(0+h, f(0+h))$. Your answer depends on h, of course.

   (c) Compute $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$ to get $f'(0)$.

   (d) Your answer to (c) is the slope of the line tangent to the graph of f at the point $(0, f(0))$. In other words, your answer is $f'(0)$. Write an equation for that tangent line.