February 16, 2011 **Name** _____

The problems count as marked. The total number of points available is 155. Throughout this test, **show your work.**

1. (10 points) The points $(2, 6)$ and $(5, 3)$ belong a line L. Find an equation for the line perpendicular to L and passing through the point $(1, 1)$.

2. (10 points) Find the exact value of $|2\pi - \sqrt{5} - 3\sqrt{2}| + 2\pi$. A decimal approximation is worth 1 point. Your answer may use radicals or symbol π.

3. (30 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to\infty} \dfrac{3x^4 - 6}{(11 - 3x^2)^2}$

   (b) $\lim_{x\to 1} \dfrac{x^4 - 1}{x^2 - 1}$

   (c) $\lim_{x\to -3} \dfrac{x^2 + 2x - 3}{x^2 + 4x + 3}$

   (d) $\lim_{x\to 2} \dfrac{\frac{1}{4x} - \frac{1}{8}}{\frac{1}{3x} - \frac{1}{6}}$

(e) $\lim_{x \to -\infty} \dfrac{|x^3|}{x^3 - x^2 + x - 1}$

(f) $\lim_{x \to 2} \dfrac{x - 2}{\sqrt{8x} - 4}$

4. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{x(8-x)}}{(x+1)(x-3)}.$$

Express your answer as a union of intervals. That is, use interval notation.

5. (12 points) Let $H(x) = (x^2-1)^2(5x+7)+(x^2-1)(5x+7)^2$. H is a polynomial of degree 5, and it has 5 zeros. Find all the zeros of H.

6. (10 points) Suppose $p(x)$ is a polynomial of degree 4 and $q(x)$ is a polynomial of degree 3. What is the degree of the polynomial $H(x) = (x^2p(x) - 1)^2 - (q(x) + x^2)^2 + x^8$? Write a sentence about your reasoning.

7. (16 points) Let

$$f(x) = \begin{cases} |x-3| & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ (4-x)^2 & \text{if } x > 2 \end{cases},$$

   (a) What is $\lim_{x\to 2^-} f(x)$?

   (b) What is $\lim_{x\to 2^+} f(x)$?

   (c) Is f continuous at $x = 2$?

   (d) What is $\lim_{x\to 1^-} f(x)$?

8. (20 points) Let $f(x) = x^2 - 2x$. Note that $f(3) = 3$

   (a) Find the slope of the line joining the points $(3, 3)$ and $(3 + h, f(3 + h))$, where $h \neq 0$. Note that $(3 + h, f(3 + h))$ is a point on the graph of f.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   (c) Replace the x with 3 in your answer to (b) to find $f'(3)$.

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(3, 3)$.

9. (20 points) For each condition listed, express in interval notation the set of all numbers that satisfy the condition. For example $1 \leq 2x - 3 < 7$ has solution the interval $[2, 5)$.

   (a) $x^2 \neq 9$

   (b) $x^2 \geq 4$

   (c) $(x - 2)(x + 3) \leq 0$

   (d) $\mid 2x + 3 \mid \geq 9$

10. (15 points) Recall that the Intermediate Value Theorem guarantees that for any function f continuous over the interval $[a, b]$ and for any number M between $f(a)$ and $f(b)$, there exists a number c such that $f(c) = M$. The function $f(x) = \frac{1}{1+\frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

    (a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.

    (b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.