October 14, 2010 **Name** _____

The problems count as marked. The total number of points available is 172. Throughout this test, **show your work.**

1. (10 points) The points $(2, k)$ and $(5, 5)$ belong to the line perpendicular to the line $3x - 2y = 7$. Find the value of k.

2. (35 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to\infty} \dfrac{3x^4 - 6}{(11 - 3x^2)^3}$

   (b) $\lim_{x\to 1} \dfrac{x^4 - 1}{x^2 - 1}$

   (c) $\lim_{h\to 0} \dfrac{(1+h)^3 - 1}{h}$.

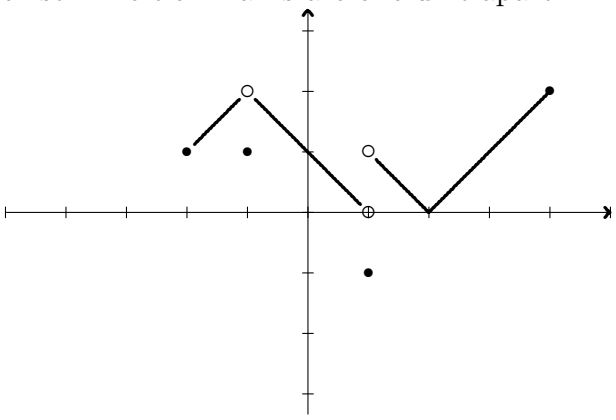
   (d) $\lim_{x\to 1} \dfrac{x^2 - 4x + 3}{x^2 + x - 2}$

   (e) $\lim_{x\to 2} \dfrac{\frac{1}{3x} - \frac{1}{6}}{\frac{1}{2x} - \frac{1}{4}}$

   (f) $\lim_{x\to -\infty} \dfrac{\sqrt{36x^2 - 3x}}{9x - 11}$

   (g) $\lim_{x\to 2} \dfrac{\sqrt{8x} - 4}{x - 2}$

3. (18 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a) $\lim_{x \to -1^-} F(x) =$

(b) $\lim_{x \to -1^+} F(x) =$

(c) $\lim_{x \to -1} F(x) =$

(d) $F(-1) =$

(e) $\lim_{x \to 1^-} F(x) =$

(f) $\lim_{x \to 1^+} F(x) =$

(g) $\lim_{x \to 1} F(x) =$

(h) $\lim_{x \to 3} F(x) =$

(i) $F(3) =$

4. (10 points) The points $(0, 0), (2, 1), (u, v)$, and $(1, -2)$ are the vertices of a square. Find u and v.

5. (12 points) Find the domain of the function

$$g(x) = \frac{\sqrt{x+1}}{(x-1)(x-3)}.$$

Express your answer as a union of intervals. That is, use interval notation.

6. (12 points) Let $H(x) = (x^2-4)^2(2x+3)^3$. Using the chain rule and the product rule,

$$H'(x) = 2(x^2-4)\cdot 2x(2x+3)^3 + (x^2-4)^2 \cdot 3(2x+3)^2 \cdot 2.$$

Three of the zeros of $H'(x)$ are $x = \pm 2$ and $x = -3/2$. Find the other two.

7. (40 points) Let $f(x) = 2x - \frac{1}{x}$ and let $g(x) = \sqrt{x^2 + 1}$. Compute, without simplifying, the composite functions listed below. Also use the product, quotient and chain rules to compute the derivatives listed.

   (a) $f \circ g(x)$

   (b) $g \circ f(x)$

   (c) $f \circ f(x)$

   (d) $g'(x)$

   (e) $f'(x)$

   (f) $\frac{d}{dx}[f \circ g(x)]$

   (g) $\frac{d}{dx}[g \cdot f(x)]$

   (h) $\frac{d}{dx}[f \div g(x)]$

8. (10 points) Suppose $p(x)$ is a polynomial of degree 3 and $q(x)$ is a polynomial of degree 4. What is the degree of the polynomial $H(x) = (x^2p(x) - 1)^2 - (q(x) + x^2)^2 + x^8$? Write a sentence about your reasoning.

9. (15 points) Let

$$f(x) = \begin{cases} |x-3| & \text{if } x < 2 \\ s & \text{if } x = 2 \\ (t-x)^2 & \text{if } x > 2 \end{cases},$$

where s and t are constants.

   (a) What is $\lim_{x\to 2^-} f(x)$?

   (b) For what values of t does $\lim_{x\to 2} f(x)$ exist.

   (c) If t is one of the values found in (b), for what value of s is f continuous at $x = 2$?

10. (10 points) The equation $x^2+4x+(y-1)^2 = 21$ is a circle. What is its radius?