February 17, 2010 **Name** ________________________________

The problems count as marked. The total number of points available is 156. Throughout this test, **show your work.**

1. (10 points) Find an equation for a line that is perpendicular to the line $2x - 3y = 7$ and which passes through the point $(4, 2)$. Write your answer in slope-intercept form.

   Solution: The slope m of the line must be the negative reciprocal of the line given. The slope of the line $2x - 3y = 7$ is $2/3$, so $m = -3/2$. Using the point-slope form of the line, we have $y - 2 = (-3/2)(x - 4)$, which simplifies to $y = -3x/2 + 8$.

2. (20 points) Let $f(x) = (2x-3)^4(5x-1)^2 + 17x^2$, and let $g(x) = (x-4)^3(8x^3) - 17x^2$.

   (a) What is the degree of the polynomial f?
   Solution: 6

   (b) What is the degree of the polynomial g?
   Solution: 6

   (c) Estimate within one unit the value of $f(1000)/g(1000)$.
   Solution: Any answer between 49 and 51 works. See the next part.

   (d) Compute $\lim_{x\to\infty} \frac{f(x)}{g(x)}$.
   Solution: $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{(2x)^4(5x)^2}{x^3 \cdot 8x^3} = \lim_{x\to\infty} \frac{400x^6}{8x^6} = 50$ because the degree of the denominator is the same as that of the numerator.

3. (15 points) Find the (implied) domain of each of the functions given below. Express your answers in interval notation.

   (a) $f(x) = \frac{1}{x^2-9}$
   Solution: Solve $x^2 - 9 = 0$ to get $x = \pm 3$. Now eliminate -3 and 3 from the number line to get $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

   (b) $g(x) = \sqrt{x-4}$
   Solution: Solve the inequality $x - 4 \geq 0$ to get $x \geq 4$, which in interval notation is $[4, \infty)$.

   (c) $h(x) = \sqrt{x(x-1)(x+3)}$
   Solution: Solve the inequality $x(x-1)(x+3) \geq 0$ using the test interval technique to get $[-3, 0] \cup [1, \infty)$.

4. (55 points) Evaluate each of the limits indicated below.

   (a) $\lim_{x\to 0} \frac{x^4-x^2}{x^2}$
   Solution: Factor and eliminate the x^2 to get

$$\lim_{x\to 0} x^2 - 1 = -1$$

   (b) $\lim_{x\to 3} \frac{\frac{1}{3x}-\frac{1}{9}}{x-3}$
   Solution: The limit of the numerator is 0 and the limit of the denominator is also zero, so we have to do some work. Find a common denominator and subtract to get $\lim_{x\to 2} \frac{\frac{3-x}{9x}}{x-3} = \lim_{x\to 3} -\frac{1}{9x} = -1/27$.

   (c) $\lim_{x\to 5} \frac{x^2-3x-10}{x-5}$
   Solution: Factor and cancel $x - 5$ to get

$$\lim_{x\to 5} x + 2 = 7$$

   (d) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$
   Solution: Rationalize the numerator to get $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{x-1}{(\sqrt{x}+1)(x-1)} = 1/2$.

   (e) $\lim_{x\to\infty} \frac{\sqrt{16x^2-3}}{11-5x}$
   Solution: $\lim_{x\to\infty} \frac{\sqrt{16x^2-3}}{11-5x} = \lim_{x\to\infty} \frac{\sqrt{16x^2/x^2-3/x^2}}{11/x-5x/x} = \frac{4}{-5} = -4/5$ because the degree of the denominator is essentially the same as that of the numerator.

For problems (f) through (k), let

$$f(x) = \begin{cases} 7-x & \text{if } x > 2 \\ 10 & \text{if } x = 2 \\ 2x+1 & \text{if } 0 \le x < 2 \\ -1 & \text{if } x < 0 \end{cases}$$

(f) $\lim_{x\to 0^-} f(x)$
Solution: -1

(g) $\lim_{x\to 0^+} f(x)$
Solution: 1

(h) $\lim_{x\to 0} f(x)$
Solution: DNE

(i) $\lim_{x\to 2^-} f(x)$
Solution: 5

(j) $\lim_{x\to 2^+} f(x)$
Solution: 5

(k) $\lim_{x\to 2} f(x)$
Solution: 5

5. (21 points) Consider the function whose properties are displayed.

a	-1	0	1	2	3	4
$\lim\limits_{x\to a^-} f(x)$	DNE	2	2	4	2	3
$\lim\limits_{x\to a^+} f(x)$	1	2	2	3	2	DNE
$f(a)$	1	2	-1	1	2	3
$\lim\limits_{x\to a^-} g(x)$	4	3	3	3	-1	0
$\lim\limits_{x\to a^+} g(x)$	1	-2	0	3	-1	DNE
$g(a)$	-1	-1	3	-3	DNE	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim\limits_{x\to 2^+} [f(x) - g(x)]$
Solution: 0

(b) $\lim\limits_{x\to 2^-} [f(x) - g(x)]$
Solution: 1

(c) $\lim\limits_{x\to 2} [f(x) - g(x)]$
Solution: DNE

(d) $(f + g)(4)$
Solution: $3 - 0 = 3$.

(e) $f \circ g \circ f(-1)$
Solution: $f \circ g \circ f(-1) = f \circ g(1) = f(3) = 2$

(f) Find all points (in the table) at which f is continuous.
Solution: $x = 0, 3$

(g) Find all points (in the table) at which g is continuous.
Solution: none

6. (10 points) Find all the x-intercepts of the function

$$g(x) = (2x^2 - 4)^2(3x + 2) + (2x^2 - 4)^3(3x + 2).$$

Solution: Factor out the common terms to get $g(x) = (2x^2 - 4)^2(3x + 2)[1 + (2x^2 - 4)] = (2x^2 - 4)^2(3x + 2)(2x^2 - 3)$. Setting each factor equal to zero, we find the zeros are $x = \pm\sqrt{2}$, $x = -2/3$ and $x = \pm\sqrt{3/2}$.

7. (25 points) Let $f(x) = \sqrt{3x-2}$. Notice that $f(6) = \sqrt{18-2} = 4$.

   (a) Find the slope of the line joining the points $(6,4)$ and $(6+h, f(6+h))$, where $h \neq 0$. Note that $(6+h, f(6+h))$ is a point on the graph of f.
   Solution: $\frac{\sqrt{3(6+h)-2}-4}{6+h-6} = \frac{\sqrt{3(6+h)-2}-4}{h}$.

   (b) Compute $f(a+h), f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.
   Solution:

   (c) Finally compute the limit as h approaches 0 to find $f'(a)$.

   Solution: Since we get zero over zero, we recall that, in this case, we should rationalize the numerator.

$$
\begin{aligned}
\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{\sqrt{3(a+h)-2}-\sqrt{3a-2}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3(a+h)-2}-\sqrt{3a-2}}{h} \cdot \frac{\sqrt{3(a+h)-2}+\sqrt{3a-2}}{\sqrt{3(a+h)-2}+\sqrt{3a-2}} \\
&= \lim_{h\to 0} \frac{3(a+h)-2-(3a-2)}{h(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \lim_{h\to 0} \frac{3h}{h(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \lim_{h\to 0} \frac{3}{(\sqrt{3(a+h)-2}+\sqrt{3a-2})} \\
&= \frac{3}{2(\sqrt{3a-2})}
\end{aligned}
$$

   (d) Replace the a with 6 to find $f'(6)$.
   Solution: $f'(6) = 3 \cdot 16^{-1/2}/2 = 3/8$

   (e) Use the information you found in part (d) to find an equation for the line tangent to f at the point $(6, f(6))$.
   Solution: Since $f(6) = 4$, we have $y - 4 = 3(x-6)/8$, which, in slope intercept form is $y = 3x/8 + 14/8$.