February 14, 2008 **Name** ____________________________________

The problems count as marked. The total number of points available is 142. Throughout this test, **show your work.**

1. (8 points) Find an equation for a line perpendicular to the line $3x - 6y = 7$ and which goes through the point $(-3, 4)$.

2. (52 points) Evaluate each of the limits indicated below.

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - 4x + 3}$

   (b) $\displaystyle\lim_{x\to 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

   (c) $\displaystyle\lim_{x\to -\infty} \frac{|\, 16x - 3\,|}{11 - 5x}$

   (d) $\displaystyle\lim_{x\to \infty} \frac{6x^2 - 3}{11 - 5x^3}$

   (e) $\displaystyle\lim_{x\to -1} \frac{x^3 + 1}{x^2 - 1}$

   (f) $\displaystyle\lim_{h\to 0} \frac{(1 + h)^3 - 1}{h}$.

For problems (g) through (m), let

$$f(x) = \begin{cases} -2 & \text{if } x < 0 \\ 2x - 2 & \text{if } 0 \leq x < 2 \\ 3 & \text{if } x = 2 \\ 7 - 2x & \text{if } x > 2 \end{cases}$$

(g) $\lim_{x \to 0^-} f(x)$

(h) $\lim_{x \to 0^+} f(x)$

(i) $\lim_{x \to 0} f(x)$

(j) $f(0)$

(k) $\lim_{x \to 2^-} f(x)$

(l) $\lim_{x \to 2^+} f(x)$

(m) $\lim_{x \to 2} f(x)$

3. (12 points) The demand curve for a certain item is given by $p = -x^2 - 2x + 100$ where x represents the quantity demanded in units of a thousand and p represents the price in dollars. The supply curve is given by $p = 8x + 25$. Find the equilibrium quantity and equilibrium price.

4. (15 points) The function $f(x) = \frac{1}{1+\frac{1}{x}}$ is continuous for all $x > 0$. Let $a = 1$.

   (a) Pick a number $b > 1$ (any choice is right), and then find a number M between $f(a)$ and $f(b)$.

   (b) Show that the conclusion to the Intermediate Value Theorem is satisfied by finding a number c in (a, b) such that $f(c) = M$.

5. (8 points) Find all the x-intercepts of the function

$$g(x) = 3(2x-5)^3(2x+1)^2 - 6(2x-5)^2(2x+1)^3.$$

6. (15 points)

   (a) Find all solutions of the equation $|| x - 3 | -5 | = 1$.

   (b) Find the (implied) domain of

$$f(x) = \sqrt{|| x - 3 | -5 | -1},$$

   and write your answer in interval notation.

7. (20 points) Let $f(x) = x^2 - x$. Note that $f(3) = 6$

   (a) Find the slope of the line joining the points $(3, 6)$ and $(3 + h, f(3 + h))$, where $h \neq 0$. Note that $(3 + h, f(3 + h))$ is a point on the graph of f.

   (b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

   (c) Replace the x with 3 in your answer to (b) to find $f'(3)$.

   (d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(3, 6)$.

8. (12 points) Given two functions,

$$g(x) = \begin{cases} 2x - 1 & \text{if } 1 < x < 4 \\ 4 - x & \text{otherwise} \end{cases} \qquad \text{and} \qquad f(x) = \begin{cases} x^2 + 3 & \text{if } x \geq 1 \\ x^2 - 4 & \text{if } x < 1 \end{cases}$$

Complete the following table.

x	$g(x)$	$f(x)$	$f \circ g(x)$	$g \circ f(x)$
-4	8	12	67	-8
-1				
0				
1				
2				
3				
3.5				
4				