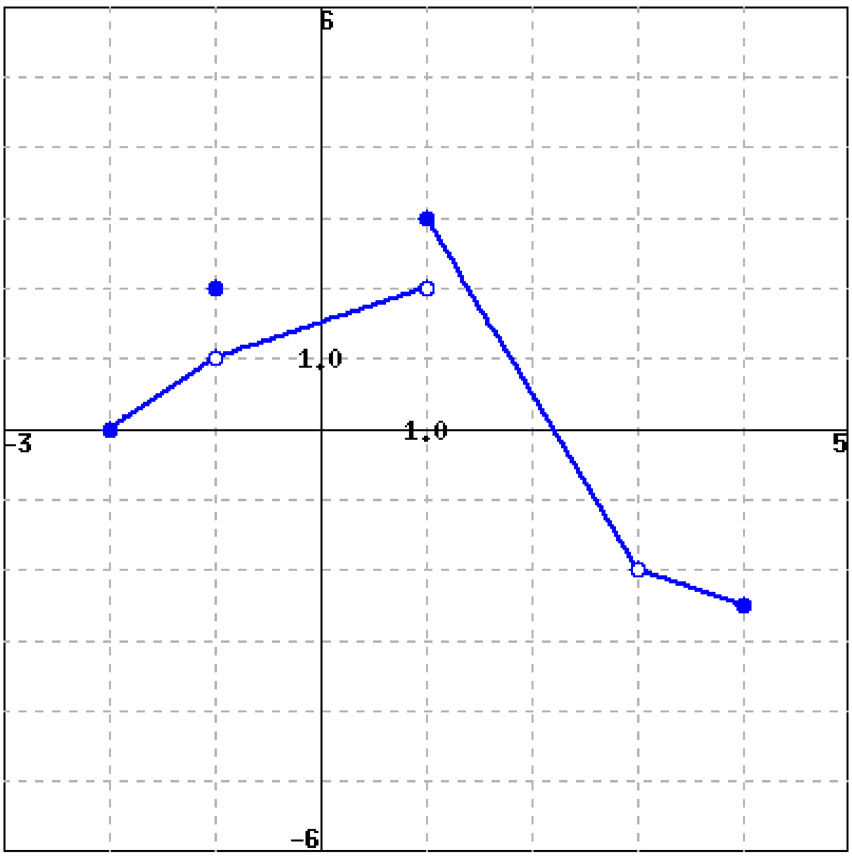
September 28, 2006 **Name** ______________________________

The problems count as marked. The total number of points available is 146. Throughout this test, **show your work.**

1. (27 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Use 'DNE' if the limit does not exist.



(a) $\lim_{x \to -1^-} F(x)$

Solution: 1

(b) $\lim_{x \to -1^+} F(x)$

Solution: 1

(c) $\lim_{x \to -1} F(x)$

Solution: 1

(d) $F(-1)$

Solution: 2

(e) $\lim_{x \to 1^-} F(x)$

Solution: 2

(f) $\lim_{x\to 1^+} F(x)$
Solution: 3

(g) $\lim_{x\to 1} F(x)$
Solution: DNE

(h) $\lim_{x\to 3} F(x)$
Solution: -2

(i) $F(3)$
Solution: DNE

2. (5 points) Evaluate the limit

$$\lim_{x\to -2} \frac{x+2}{x^2-3x-10}$$

Solution: Factor and cancel the $x+2$s to get

$$\lim_{x\to 2} \frac{1}{x-5} = -1/7$$

3. (5 points) Evaluate the limit

$$\lim_{x\to 1} \frac{x^4-x^2}{x^2-1}$$

Solution: Factor and eliminate the x^2-1 to get

$$\lim_{x\to 1} x^2 = 1$$

4. (5 points) Evaluate the limit

$$\lim_{x\to 1} \frac{\frac{1}{x}-\frac{1}{1}}{x-1}$$

Solution: Do the fraction arithmetic to get limit $= -1$.

5. (18 points)

$$f(x) = \begin{cases} 13 & \text{if } x > 8 \\ 10 & \text{if } x = 8 \\ -x+13 & \text{if } 0 \le x < 8 \\ 13 & \text{if } x < 0 \end{cases}$$

Sketch the graph of this function and find following limits if they exist (if not, enter DNE).

(a) $\lim_{x\to 8^-} f(x)$
Solution: 5

(b) $\lim_{x\to 8^+} f(x)$
Solution: 13

(c) $\lim_{x\to 8} f(x)$
Solution: DNE

(d) $\lim_{x\to 0^-} f(x)$
Solution: 13

(e) $\lim_{x\to 0^+} f(x)$
Solution: 13

(f) $\lim_{x\to 0} f(x)$
Solution: 13

Solution:

6. (16 points) Consider the function whose properties are displayed.

a	-1	0	1	2	3	4
$\lim_{x\to a^-} f(x)$	DNE	1	1	4	2	3
$\lim_{x\to a^+} f(x)$	1	2	1	3	2	DNE
$f(a)$	1	2	-1	1	2	3
$\lim_{x\to a^-} g(x)$	4	1	3	3	1	0
$\lim_{x\to a^+} g(x)$	1	2	0	3	1	DNE
$g(a)$	1	-1	3	3	DNE	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim_{x \to 3} [f(x) + g(x)]$
Solution: 3

(b) $f(1)g(1)$
Solution: -3

(c) Find all points (in the table) at which g is continuous.
Solution: $x = 2$

(d) Find all points (in the table) at which f is continuous.
Solution: $x = 3$

7. (10 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{4x - 3}$ and let $[a, b] = [1, 7]$. Finally, suppose $M = 2$. Find the number c whose existence is guaranteed by IVT.

   Solution: We need to solve the equation $\sqrt{4x - 3} = 2$ for x. Square both sides to get $4x - 3 = 4$, from which it follows that $x = 7/4$.

8. (8 points) Evaluate the limit

$$\lim_{x \to \infty} \frac{\sqrt{4x^2 - 3}}{11 - 10x}$$

   Solution: Divide the numerator and denominator by x to get $\lim_{x \to \infty} \frac{\sqrt{4x^2 - 3}}{11 - 10x} = \lim_{x \to \infty} \frac{\sqrt{4x^2/x^2 - 3/x^2}}{11/x - 10x/x} = 2/(-10) = -1/5$ because the degree of the denominator is the same as that of the numerator.

9. (6 points) Compute the exact value of $|8\pi - 20\sqrt{2}| + |8\pi - 25| - |6\sqrt{2} - 10|$. No points for a decimal approximation.

   Solution: $|8\pi - 20\sqrt{2}| + |8\pi - 25| - |6\sqrt{2} - 10| = 20\sqrt{2} - 8\pi + 8\pi - 25 - 6\sqrt{2} - 10 = 26\sqrt{2} - 35$, because $8\pi - 20\sqrt{2} < 0$, $8\pi - 25 > 0$ and $6\sqrt{2} - 10 < 0$.

10. (15 points) Suppose

$$f(x) = \begin{cases} x - 3 & \text{if } x < 1 \\ 2x + 2 & \text{if } x > 1 \end{cases}$$

   and $g(x) = x^2 + 5$. Find the two composite functions

(a) $f \circ g(x)$
Solution: $f \circ g(x) = 2x^2 + 12$.

(b) $g \circ f(x)$
Solution: $g \circ f(x) = \begin{cases} x^2 - 6x + 14 & \text{if } x < 1 \\ 4x^2 + 8x + 9 & \text{if } x > 1 \end{cases}$.

11. (6 points) Find the (implied) domain of

$$f(x) = \frac{\sqrt{x-2}}{x^2 - 9},$$

and write your answer in interval notation.

Solution: The domain D includes all real numbers greater than or equal to 2 except 3, which must be eliminated because it makes the denominator zero. Thus, $D = [2, 3) \cup (3, \infty)$.

12. (15 points) Let $f(x) = \sqrt{2x+1}$.

(a) Find the slope of the line joining the points $(4, 3)$ and $(x, f(x))$, where $x \neq 4$.
Solution: $\frac{\sqrt{2x-1}-3}{x-5}$.

(b) Compute $f(a+h), f(a)$, and finally $\frac{f(a+h)-f(a)}{h}$.
Solution:

(c) Replace the a with 4 and take the limit as h approaches 0. You have just found $f'(4)$.
Solution:

(d) Use the information found in (c) to write an equation for the line tangent to the graph of f at the point $(4, 3)$.
Solution: $y - 3 = (1/3)(x - 5)$, so $y = x/3 + 4/3$.

13. Bonus problem. (10 points) How many points in the plane satisfy both

a. $|x| = 4$ and
b. $x^2 - 2x + y^2 - 14y = -25$.

Solution: The answer is 3. Complete the square to see that the second equation is a circle $(x-1)^2 + (y-7)^2 = 5^2$, so the points for which $|x| = 4$ can be split into two cases:

(a) $x = -4$: $(-5)^2 + (y-7)^2 = 25 \Rightarrow (y-7)^2 = 0 \Rightarrow y = 7$ and

(b) $x = 4$: $(3)^2 + (y-7)^2 = 25 \Rightarrow (y-7)^2 = 16 \Rightarrow y = 3$ or $y = 11$.