July 14, 2005 **Name** _____

The first 6 problems count 6 points each and the rest count as marked. The total number of points available is 137. Throughout this test, **show your work.**

1. What is the degree of the polynomial $p(x) = (x^2 - 1)^3(x^5 - 7)$?

2. Let P denote the midpoint of the line segment joining $(4, 3)$ and $(-6, 9)$. What is the distance from P to the point $(0, 3)$?

3. Compute the exact value of $|4\pi - 5\sqrt{2}| + |4\pi - 13| - |5\sqrt{2} - 8|$.

4. Find the (implied) domain of

$$f(x) = \frac{\sqrt{x-6}}{(x-2)(x-9)},$$

and write your answer in interval notation.

5. Find all the x-intercepts of the function

$$t(x) = (2x-1)^3(x-1)^2 - (2x-1)^2(x-1)^3.$$

6. Find an equation for a line perpendicular to the line $3x - 4y = 7$ and which goes through the point $(-2, -5)$.

7. (8 points) The line tangent to the graph of $y = e^{4x}$ at the point $(0, 1)$ has slope 4. What is the x-intercept of the line? Hint: recall the x-intercept is the point where the line crosses the x-axis.

8. (48 points) Compute each of the following limits.

(a) Let $f(x) = \begin{cases} x+2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ 8 - x^2 & \text{if } x > 2 \end{cases}$

$\lim_{x \to 2} f(x)$

(b) $\lim_{x \to 0} \dfrac{x^2 - 3x}{x}$

(c) $\lim_{x \to 3} \dfrac{x^2 - 3x}{x^2 + x - 12}$

(d) $\lim_{x \to 1} \dfrac{x^2 - 1}{x^3 - 1}$

(e) $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

(f) $\lim_{x \to 1} \dfrac{\frac{1}{3x} - \frac{1}{3}}{x - 1}$

(g) $\lim_{h \to 0} \dfrac{(3+h)^3 - 27}{h}$. Hint: you will have to work out the expanded form of $(3+h)^3$.

(h) $\lim_{x \to \infty} \dfrac{3x^2}{(1-2x)^2}$

9. (15 points) Let $k(x) = x^2 - x$. Evaluate and simplify $\frac{k(x+h)-k(x)}{h}$. Then find the limit of the expression as h approaches 0.

10. (30 points) Consider the rational function $r(x) = \frac{(x+1)^2(2x+5)}{4x^3-16x}$.

    (a) Estimate the value $r(1000)$. Does $r(x)$ have a horizontal asymptote? Determine the degrees of the numerator n and the denominator m.

    (b) Factor the denominator completely. Determine the vertical asymptotes.

    (c) Use the Test Interval Technique to solve the inequality $r(x) \geq 0$. Be sure to show your work, including the matrix of values of the factors at the test points.