Math 1120   **Calculus**   **Test 1**

September 24, 2004   **Name** _____

The first 6 problems count 6 points each and the rest count as marked. The total number of points available is 137. Throughout this test, **show your work.**

1. What is the degree of the polynomial $p(x) = (x^3 - 1)^3(x^2 + 7)$?

2. Let P denote the midpoint of the line segment joining $(2, 3)$ and $(-6, 9)$. What is the distance from P to the point $(0, 3)$?

3. Compute the exact value of $|\pi + \sqrt{3} - 5| - |2\pi - \sqrt{3} + 1|$.

4. Find the (implied) domain of

$$f(x) = \frac{\sqrt{x - 2}}{(x - 5)(x - 7)},$$

   and write your answer in interval notation.

5. Find all the x-intercepts of the function

$$t(x) = (2x - 1)^3(x - 1)^2 - (2x - 1)^2(x - 1)^3.$$

6. Find an equation for a line perpendicular to the line $3x - 4y = 7$ and which goes through the point $(-2, -3)$.

7. (8 points) The line tangent to the graph of $y = e^{3x}$ at the point $(0, 1)$ has slope 3. What is the x-intercept of the line?

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8. (48 points) Compute each of the following limits.

   (a) Let $f(x) = \begin{cases} x+2 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 4-x^2 & \text{if } x > 1 \end{cases}$

   $\displaystyle\lim_{x \to 1} f(x)$

   (b) $\displaystyle\lim_{x \to 0} \frac{x^2 - 2x}{x}$

   (c) $\displaystyle\lim_{x \to 3} \frac{x^2 - 3x}{x^2 + x - 12}$

   (d) $\displaystyle\lim_{x \to 2} |x^2 - \sqrt{16x - 7}|$

   (e) $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1}$

   (f) $\displaystyle\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

   (g) $\displaystyle\lim_{x \to 1} \frac{\frac{1}{3x} - \frac{1}{3}}{x - 1}$

   (h) $\displaystyle\lim_{x \to \infty} \frac{2x^2}{(1 - x)^2}$

9. (15 points) Let $k(x) = x^2 - x$. Evaluate and simplify $\frac{k(x+h) - k(x)}{h}$. Then find the limit of the expression as h approaches 0.

10. (30 points) Consider the rational function $r(x) = \frac{(x+1)^2(2x+5)}{4x^3-16x}$.

   (a) Estimate the value $r(1000)$. Does $r(x)$ have a horizontal asymptote? Determine the degrees of the numerator n and the denominator m.

   (b) Factor the denominator completely. Determine the vertical asymptotes.

   (c) Use the Test Interval Technique to solve the inequality $r(x) \geq 0$. Be sure to show your work, including the matrix of values of the factors at the test points.