

February 25, 2019

Name _____

The problems count as marked. The total number of points available is 155.

Throughout this test, **show your work.**

1. (6 points) Find an equation in slope-intercept form for a line perpendicular to the line $3x - 6y = 7$ and which goes through the point $(-3, 5)$.

2. (20 points) The equations $x^2 + 2x + y^2 = 15$ and $x^2 - 10x + y^2 - 16y = -53$ are both circles.
 - (a) (8 points) Use the ‘complete the square’ idea to find the centers and radii of the circles.

 - (b) Find the distance between the centers.

 - (c) Find the midpoint of the line segment joining the centers.

 - (d) Find the slope of the line joining the centers.

 - (e) Do the circles have one, two, or no points in common? Write a complete sentence to justify your answer.

3. (42 points) Evaluate each of the limits (and function values) indicated below.

$$(a) \lim_{x \rightarrow 2} \frac{(x+1)^2 - 9}{x-2}$$

$$(b) \lim_{x \rightarrow 2} \frac{2-x}{\frac{1}{2x} - \frac{1}{4}}$$

$$(c) \lim_{x \rightarrow 6} \frac{\sqrt{2x-3} - 3}{x-6}$$

$$(d) \lim_{x \rightarrow -1} \frac{x^3 + 6x^2 + 11x + 6}{x^3 - 4x^2 + x + 6}$$

$$(e) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1}$$

$$(f) \lim_{h \rightarrow \infty} \frac{(2x^2 - 5)(3x + 1)}{4x^3 + x^2 - 17}.$$

4. (18 points) Let

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x - 1 & \text{if } 0 \leq x < 2 \\ -1 & \text{if } x = 2 \\ 1 & \text{if } 2 < x \leq 7 \end{cases}$$

Find the value, if it exists, of each item below. Use DNE when the limit does not exist.

(a) What is the domain of f ?

(b) $\lim_{x \rightarrow 0^-} f(x)$

(c) $\lim_{x \rightarrow 0^+} f(x)$

(d) $\lim_{x \rightarrow 0} f(x)$

(e) $f(0)$

(f) $\lim_{x \rightarrow 2^-} f(x)$

(g) $\lim_{x \rightarrow 2^+} f(x)$

(h) $\lim_{x \rightarrow 2} f(x)$

(i) $f(2)$

5. (10 points) Find all the x -intercepts of the function

$$g(x) = (2x^2 - 1)^2(3x + 1) - (2x^2 - 1)(3x + 1)^2.$$

6. (15 points)

- (a) Find all solutions of the inequality $|2x - 7| \leq 5$ and write your solution in interval notation.

- (b) Find the (implied) domain of

$$f(x) = \sqrt{|2x - 7| - 3},$$

and write your answer in interval notation.

7. (24 points) Compute the following derivatives.

(a) Let $f(x) = \frac{x^2-2x}{3x-x^2}$. Find $\frac{d}{dx}f(x)$.

(b) Let $g(x) = \sqrt{x^3 + 2x + 4}$. What is $g'(x)$?

(c) Find $\frac{d}{dx}((3x + 1)^2 \cdot (4x^2 - 1))$

(d) Let $f(x) = (2x^2 + 1)^4$. Find $f'(x)$.

8. (20 points) Let $f(x) = \frac{1}{x+1}$. Note that $f(0) = 1$.

(a) Find the slope of the line joining the points $(0, 1)$ and $(0 + h, f(0 + h)) = (h, f(h))$, where $h \neq 0$.

(b) Evaluate and simplify $\frac{f(x+h)-f(x)}{h}$. Then find the limit of the expression as h approaches 0.

(c) Replace the x with 0 in your answer to (b) to find $f'(0)$.

(d) Use the information given and that found in (c) to find an equation for the line tangent to the graph of f at the point $(0, 1)$.