February 6, 2002 Your name ____________________

The first 11 problems count 5 points each and the final ones counts as marked. Problems 1 through 11 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 1 through 11, but you must show your work on the other problems. The total number of points available is 127.

1. What is the coefficient of the x^2 term in the product $(x^2 - 3x + 7)(x + 5)$?

   (A) -3 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

   Solution: B. The product is $(x^2 - 3x + 7)(x + 5) = x^3 + 2x^2 - 8x + 35$, so the answer is 2.

2. Which of the following is a factor of $8x^3 - y^3$? Circle all those that apply.

   (A) $x - y$ **(B)** $x + y$ **(C)** $2x - y$ **(D)** $2x + y$ **(E)** $4x - y$

   Solution: C. This factors as a difference of two cubes: $8x^3 - y^3 = (2x)^3 - y^3 = (2x - y)((2x)^2 + (2x)y + y^2)$, so the factor in question is $2x - y$.

3. Which of the following is a root of $x^2 - 2x = 15$?

   (A) 2 **(B)** 3 **(C)** 5 **(D)** 15 **(E)** 17

   Solution: C. The equation is equivalent to $x^2 - 2x - 15 = 0$ which is the same as $(x - 5)(x + 3) = 0$, so the root are $x = 5$ and $x = -3$.

4. How many roots does the equation below have?

$$x^2(x^2 - 3) - 4(x^2 - 3) = 0$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

   Solution: E. Factor to get $(x^2 - 3)(x^2 - 4) = (x - \sqrt{3})(x + \sqrt{3})(x - 2)(x + 2)$ which has four zeros.

5. What is the distance between the point $(-2, 3)$ and the midpoint of the line segment joining $(3, 9)$ and $(5, 13)$?

   (A) $\sqrt{14}$ **(B)** 9 **(C)** $\sqrt{90}$ **(D)** 10 **(E)** $\sqrt{149}$

   Solution: D. The midpoint is the point $(4, 11)$ so the distance is $\sqrt{(4 + 2)^2 + (11 - 3)^2} = \sqrt{100} = 10$.

6. What is the value of $|6\pi - 19| - |16 - 5\pi|$?

   (A) $3 + \pi$ **(B)** $\pi - 3$ **(C)** $3 - \pi$ **(D)** $35 + 11\pi$ **(E)** $11\pi - 35$

   Solution: C. By the definition of absolute value, $|6\pi - 19| - |16 - 5\pi| = 19 - 6\pi - (16 - 5\pi) = 3 - \pi$.

7. If $b^2 - 4ac = 0$, then the number of roots of $ax^2 + bx + c = 0$ is

   (A) 0 **(B)** 1 **(C)** 2

   (D) 3 **(E)** cannot be determined from this information

   Solution: B. By the quadratic formula, the two root are $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$, which are identical when $b^2 - 4ac = 0$, so the answer is 1.

8. Which of the following points belongs to the circle of radius 5 and center at $(4, 7)$?

   (A) $(7, 8)$ **(B)** $(7, 9)$ **(C)** $(7, 10)$ **(D)** $(7, 11)$ **(E)** $(7, 12)$

   Solution: D. The point must be 5 units from $(4, 7)$, so it satisfies $\sqrt{(7-4)^2 + (y-7)^2} = 5$, which simplifies to $(y - 7)^2 = 25 - 9 = 16$ which implies that $|y - 7| = 4$. Thus either $y = 3$ or $y = 11$.

9. What is the slope of the line passing through $(7, 8)$ that is perpendicular to the line $3x - 4y = 7$?

   (A) $3/4$ **(B)** $-3/4$ **(C)** $4/3$ **(D)** $-4/3$ **(E)** $3/7$

   Solution: D. The slope of $3x - 4y = 7$ is $3/4$, so any perpendicular line has slope $-4/3$.

10. What is the slope of the line that includes the points $(-2, 3)$ and $(3, -4)$?

    (A) $-7/5$ **(B)** $-1/5$ **(C)** $1/5$ **(D)** $7/5$ **(E)** 7

    Solution: A. The slope is $\frac{-4-3}{3-(-2)} = \frac{-7}{5}$.

11. Which of the following points is not in the domain of the function f defined by $f(x) = \sqrt{(x-1)(x+1)}$?

    (A) -3 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 4

    Solution: C. The domain is the set of x for which $(x - 1)(x + 1) \geq 0$. The only value in the list for which this fails is 0.

On all the following questions, **show your work.**

12. (12 points) The relationship between the Celsius (C) and the Fahrenheit (F) temperature scales is linear. Water boils at $212°F$ which is equivalent to $100°C$. Also, water freezes at $32°F$ and at $0°C$. Find F as a function of C and use this equation to find the Fahrenheit temperature in the central square in Seville, Spain, in August, 1997 when the Celsius temperature was 53°.

    Solution: We are trying to find a linear function which include the two points $(0, 32)$ and $(100, 212)$. The slope is $m = \frac{212-32}{100-0} = 9/5$, so the point slope form gets the job done: $F - 32 = (9/5)(C - 0)$ which is equivalent to $F = 9C/5 + 32$. The easily remembered rule, which works pretty well for intermediate temperatures is "double and then add 30". The Fahrenheit temperature at $C = 53$ is $9 \cdot 53/5 + 32 = 127.4°$. Yes, it was a very hot week!

13. (20 points) Let $f(x) = \sqrt{2x}$. Use the definition of derivative (ie, the difference quotient) to compute $f'(x)$.

    Solution: We need to find the limit of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as h approaches 0. Thus, $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}$. To evaluate this limit, we must rationalize the numerator by multiplying by the number 1 written in a special form $\frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}}$. The result is $\frac{2(x+h)-2x}{h(\sqrt{2(x+h)}+\sqrt{2x})}$. Do the arithmetic to get $\lim_{h\to 0} \frac{2h}{h(\sqrt{2(x+h)}+\sqrt{2x})} = \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)}+\sqrt{2x}} = \frac{1}{\sqrt{2x}}$.

14. (20 points) Use the definition of derivative (ie, the difference quotient) to compute the derivative of the following function: $f(x) = 1/x$.

    Solution: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{x-x-h}{x(x+h)h} = -\frac{1}{x^2}$.