

September 18, 2001

Your name _____

The first 6 problems count 4 points each and the final ones counts as marked. Problems 1 through 6 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 1 through 6, but you must show your work on the other problems. The total number of points available is 125.

1. Which of the following is a factor of $x^4 - x$? Circle all those that apply.

(A) x (B) $x - 1$ (C) $x + 1$ (D) $x^2 + x + 1$ (E) $x^2 - x + 1$

2. How many roots does the equation below have?

$$x(x^2 - 3) - 4(x^2 - 3) = 0$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

- 3.

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} =$$

(A) $\frac{x+1}{x-1}$ (B) $\frac{x-1}{x+1}$ (C) $x-1$ (D) $1-x$ (E) x

4. What is the radius of the circle whose equation is given by $x^2 - 8x + y^2 + 6y = 24$?

(A) 4 (B) $\sqrt{24}$ (C) 5 (D) 6 (E) 7

5. Which of the following is a solution to $2(5 - 3x) - 2 \cdot 5 - 3x = 108$? Circle all that apply.

(A) none (B) -12 (C) -9 (D) -2 (E) 0

6. Which of the following is not a solution to $3(x-2)^3(x+1)^2 - 2(x-2)^2(x+1)^3 = 0$? Circle all that apply.

(A) -2 (B) -1 (C) 0 (D) 2 (E) 8

On all the following questions, **show your work**.

7. (7 points) Find all roots of the equation

$$(x - 1)(x + 1) + (x - 2)(x + 1) = 0.$$

8. (7 points) Rationalize the numerator of the expression $\frac{\sqrt{4+h}-2}{h}$, and express your answer in simplified form.

9. (7 points) Find a complete factorization of $x^6 - 64$.

10. (7 points) Find a symbolic representation of $f \circ g(x)$ in the case where $f(x) = \sqrt{2x} - 5$ and $g(x) = 7 - x$. Then find the implied domain of $f \circ g(x)$

11. (7 points) The points $A = (0, 0)$, $B = (8, 0)$, and $C = (3, 6)$ are the vertices of triangle. Find the length of the longest side.

12. (7 points) What is the slope of the line joining the points $(-2, f(-2))$ and $(4, f(4))$, where f is the function defined by

$$f(x) = \begin{cases} x^2 - |x| & \text{if } x \leq 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$

13. (7 points) Find the (implied) domain of the function $f(x) = \frac{\sqrt{x}}{x-3}$.

14. (12 points) Suppose the functions f and g are given by the table of values shown. Complete the table by calculating the values of $f \circ g(x)$ and $g \circ f(x)$ for each of the values of x in the table.

x	$f(x)$	$g(x)$	$f \circ g(x)$	$g \circ f(x)$
0	2	1		
1	3	5		
2	2	1		
3	5	4		
4	4	3		
5	2	0		

15. (40 points) Evaluate each of the limits, or state that it does not exist.

$$(a) \lim_{x \rightarrow \infty} \frac{x^2 + 9x - 11}{2x^2 - 4x + 23}$$

$$(b) \lim_{z \rightarrow 2} \frac{z^3 - 8}{z - 2}$$

$$(c) \lim_{h \rightarrow 3} \frac{(2 - h)^2 + (2 + h)^2}{h^2 - 3h + 6}$$

$$(d) \lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 9}$$

$$(e) \lim_{x \rightarrow 2} f(x)$$

where

$$f(x) = \begin{cases} (x - 4)^2 & \text{if } x < 2 \\ 7 & \text{if } x = 2 \\ 5x - 6 & \text{if } x > 2 \end{cases}$$