October 2, $2014 \quad$ Name
The problems count as marked. The total number of points available is 153 . Throughout this test, show your work.

1. (10 points) Find all solutions to $||2 x-15|-3|=2$.
2. (24 points) The set of points $C_{1}$ in the plane satisfying $x^{2}+y^{2}-4 y=0$ is a circle. The set $C_{2}$ whose points satisfy $x^{2}-24 x+y^{2}-14 y=-49$ is also a circle.
(a) What is the distance between the centers of the circles?
(b) How many points in the plane belong to both circles. That is, how many points in the plane satisfy both equations?
(c) Find an equation for the line connecting the centers of the circles.
3. (35 points) Evaluate each of the limits indicated below.
(a) $\lim _{x \rightarrow \infty} \frac{2 x^{6}-6}{\left(11-2 x^{2}\right)^{3}}$
(b) $\lim _{x \rightarrow 1} \frac{x^{4}-1}{x^{2}-1}$
(c) $\lim _{h \rightarrow 0} \frac{(2+h)^{3}-8}{h}$.
(d) $\lim _{x \rightarrow 1} \frac{x^{2}-4 x+3}{x^{2}+x-2}$
(e) $\lim _{x \rightarrow 2} \frac{\frac{1}{3 x}-\frac{1}{6}}{\frac{1}{2 x}-\frac{1}{4}}$
(f) $\lim _{x \rightarrow-\infty} \frac{\sqrt{36 x^{2}-3 x}}{9 x-11}$
(g) $\lim _{x \rightarrow 2} \frac{\sqrt{8 x}-4}{x-2}$
4. (12 points) Find the domain of the function

$$
g(x)=\frac{\sqrt{(x+10)(2 x-3)(3 x-17)}}{x^{2}-4} .
$$

Express your answer as a union of intervals. That is, use interval notation.
5. (12 points) Let $H(x)=\left(x^{2}-4\right)^{2}(2 x+3)^{3}$. Using the chain rule and the product rule,

$$
H^{\prime}(x)=2\left(x^{2}-4\right) \cdot 2 x(2 x+3)^{3}+\left(x^{2}-4\right)^{2} \cdot 3(2 x+3)^{2} \cdot 2 .
$$

Three of the zeros of $H^{\prime}(x)$ are $x= \pm 2$ and $x=-3 / 2$. Find the other two.
6. (25 points) Given two functions,

$$
g(x)=2 x+1
$$

and

$$
f(x)=\left\{\begin{array}{cl}
x^{2} & \text { if } x \leq 2 \\
-2 & \text { if } x>3
\end{array}\right.
$$

Use 'dne' for 'does not exist.'
(a) Write the domain of $f$ in interval notation.
(b) Compute $\lim _{x \rightarrow 3^{+}} f(x)$
(c) Compute $\lim _{x \rightarrow 3^{-}} f(x)$
(d) Complete the following table.

| $x$ | $g \circ f(x)$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| $\pi$ |  |

(e) Find the symbolic representation of $g \circ f(x)$
7. (25 points) Let $f(x)=\sqrt{4 x-3}$.
(a) Let $h$ be a positive number. What is the slope of the line passing through the points $(3, f(3))$ and $(3+h, f(3+h))$. Your answer depends on $h$, of course. Suppose your answer is called $G(h)$.
(b) Compute $\lim _{h \rightarrow 0} G(h)$.
(c) Your answer to (b) is the slope of the line tangent to the graph of $f$ at the point $(3, f(3))$. In other words, your answer is $f^{\prime}(3)$. Write and equation for the tangent line.
8. (10 points) Evaluate the following limits.
(a) $\lim _{x \rightarrow \infty} \frac{(2-x)(10+6 x)}{(3-5 x)(8+8 x)}$
(b) $\lim _{x \rightarrow-\infty} \frac{(2-x)(10+6 x)}{(3-5 x)(8+8 x)}$

