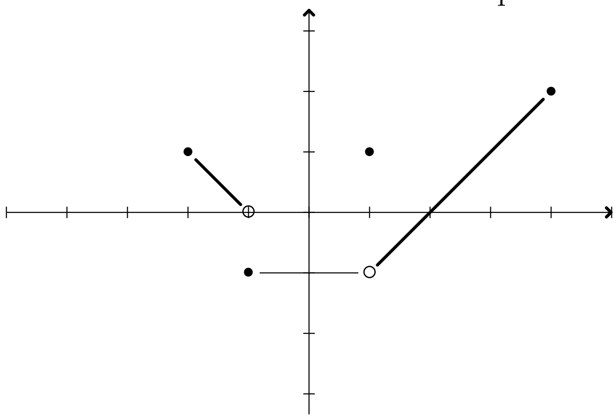
February 9, 2006 **Name** ____________________

The problems count as marked. The total number of points available is 161. Throughout this test, **show your work.**

1. (18 points) Consider the function F whose graph is given below. Evaluate each of the following expressions. Note: Enter 'DNE' if the limit does not exist. The tick marks are one unit apart.



(a) $\lim_{x \to -1^-} F(x) = 0$

(b) $\lim_{x \to -1^+} F(x) = -1$

(c) $\lim_{x \to -1} F(x) = DNE$

(d) $F(-1) = -1$

(e) $\lim_{x \to 1^-} F(x) = -1$

(f) $\lim_{x \to 1^+} F(x) = -1$

(g) $\lim_{x \to 1} F(x)$ =-1

(h) $\lim_{x \to 3} F(x) = 1$

(i) $F(3) = 1$

2. (6 points) Evaluate the limit

$$\lim_{x \to -7} \frac{x^2 + 8x + 7}{x + 7}$$

Solution: Factor and cancel the $x + 7$ to get

$$\lim_{x \to -7} x + 1 = -6$$

3. (6 points) Evaluate the limit

$$\lim_{x \to 2} \frac{x - 2}{x^2 + 3x - 10}$$

Solution: Factor and cancel the $x - 2$ to get

$$\lim_{x \to 2} \frac{1}{x + 5} = 1/7$$

4. (6 points) Evaluate the limit

$$\lim_{x \to 1} \frac{x^4 - 1}{x^2 - 1}$$

Solution: Factor and eliminate the $x^2 - 1$ to get

$$\lim_{x \to 1} x^2 + 1 = 2$$

5. (6 points) Evaluate the limit

$$\lim_{t \to 9} \frac{9 - t}{3 - \sqrt{t}}$$

Solution: Rationalize the denominator to get

$$\lim_{t \to 9} \frac{(9 - t)(3 + \sqrt{t})}{9 - t} = 6$$

6. (6 points) Evaluate the limit

$$\lim_{x \to 4} \frac{\frac{1}{x} - \frac{1}{4}}{x - 4}$$

Solution: Do the fraction arithmetic, noting that $4 - x = -(x - 4)$, so the limit is $-1/16$.

7. (8 points) Find the midpoint of the segment joining $(6, 3)$ and $(-2, 7)$. Then find the distance from that midpoint to the point $(1, 0)$.

   Solution: The midpoint of (x_1, y_1) and (x_2, y_2) is $((x_1 + x_2)/2, (y_1 + y_2)/2)$, so the midpoint in question is $(6 - 2)/2, (3 + 7)/2) = (2, 5)$, and its distance to the origin is $\sqrt{1^2 + 5^2} = \sqrt{26}$.

8. (8 points) Let a polynomial be defined by $p(x) = (2x - 3)^4(x - 1)(3x + 5)^3$. What is the degree of p? When p is written in standard form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n \neq 0$, what is a_8? What is a_0?

   Solution: The product of the three factors has degree $n = 4+1+3 = 8$. The first factor is $(2x-3)^4 = 16x^4 + \ldots$ while the third one is $(3x+5)^3 = 27x^3 + \ldots$. Therefore the coefficient a_8 of x^8 is just the the product $16 \cdot 27 = 432$ and $a_0 = -10125$.

9. (18 points) Let

$$f(x) = \begin{cases} 9 & \text{if } x < -5 \\ -2x + 8 & \text{if } -5 \le x < 2 \\ 0 & \text{if } x = 2 \\ 4 & \text{if } x > 2 \end{cases}$$

   Sketch the graph of this function and find following limits if they exist (if not, enter DNE).

   (a) $\lim_{x \to 2^-} f(x) = 4$

   (b) $\lim_{x \to 2^+} f(x) = 4$

   (c) $\lim_{x \to 2} f(x) = 4$

   (d) $\lim_{x \to -5^-} f(x) = 9$

   (e) $\lim_{x \to -5^+} f(x) = 18$

   (f) $\lim_{x \to -5} f(x)\ DNE$

10. (12 points) Consider the function whose properties are displayed.

a	-1	0	1	2	3	4
$\lim_{x\to a^-} f(x)$	DNE	1	1	3	2	3
$\lim_{x\to a^+} f(x)$	1	1	1	3	2	DNE
$f(a)$	1	1	-1	3	2	3
$\lim_{x\to a^-} g(x)$	4	1	3	3	1	0
$\lim_{x\to a^+} g(x)$	1	2	3	3	1	DNE
$g(a)$	1	-1	3	3	1	0

Using the table above calculate the limits below. Enter 'DNE' if the limit doesn't exist OR if limit can't be determined from the information given.

(a) $\lim_{x\to -1^-}[f(x)+g(x)]$
Solution: DNE

(b) $\lim_{x\to 3}[f(x)+g(x)]$
Solution: 3

(c) $f(1)g(1)$
Solution: -3

(d) $f(2)+g(0)$
Solution: 2

(e) Find all points (in the table) at which f is discontinuous.
Solution: $x=-1,1,4$

11. (6 points) Evaluate the limit

$$\lim_{x\to\infty}\frac{2+4x}{9-2x}$$

Solution: By the asymptote theorem, $y=4/(-2)=-2$ is the horizontal asymptote.

12. (6 points) Evaluate the limit

$$\lim_{x\to\infty} \frac{2x^3 - 10x^2 - 3x}{7 - 6x - 10x^4}$$

Solution: Similarly,

$$\lim_{x\to\infty} \frac{2x^3 - 10x^2 - 3x}{7 - 6x - 10x^4} = \lim_{x\to\infty} \frac{2x^3}{-10x^4} = 0$$

, because the degree of the denominator is larger than that of the numerator.

13. (8 points) Find the (implied) domain of

$$f(x) = \frac{\sqrt{x-7}}{(x-2)(x-9)},$$

and write your answer in interval notation.

Solution: The domain D includes all real numbers greater than or equal to 7 except 9, which must be eliminated because it makes the denominator zero. Thus, $D = [7, 9) \cup (9, \infty)$.

14. (8 points) Find all the x-intercepts of the function

$$g(x) = 3(2x - 7)^3(2x + 1)^2 - 6(2x - 7)^2(2x + 1)^3.$$

Solution: Factor the common stuff out to get $3(2x - 1)^2(x - 1)^2[2x - 7 - 2(2x + 1)]$. Setting each of the three factors to zero yields $x = -1/2, x = 7/2$, and $x = -9/2$.

15. (8 points) Compute the exact value of $|6\pi - 10\sqrt{2}| + |6\pi - 20| - |5\sqrt{2} - 8|$. No points for a decimal approximation.

Solution: $|6\pi-10\sqrt{2}|+|6\pi-20|-|5\sqrt{2}-8| = 6\pi-10\sqrt{2}+20-6\pi-[8-5\sqrt{2}] = 12 - 5\sqrt{2}$, because $6\pi - 10\sqrt{2} > 0$, $6\pi - 20 < 0$ and $5\sqrt{2} - 8 < 0$. Thus $|6\pi - 10\sqrt{2}| = 10\sqrt{2} - 6\pi$, $|6\pi - 20| = 20 - 6\pi$, and $-|5\sqrt{2} - 8| = -(8 - 5\sqrt{2})$.

16. (8 points) Find an equation for a line perpendicular to the line $2x - 5y = 11$ and which goes through the point $(-2, 6)$.

Solution: The given line has slope $2/5$ so the one perpendicular has slope $-5/2$. Hence $y - 6 = (-5/2)(x + 2)$. Thus $y = -5x/2 + 1$.

17. (8 points) Suppose $f(x) = \sqrt{3x-1}$ and $g(x) = x^2+4$. Find the two composite functions

   (a) $f \circ g(x)$
   Solution: $f \circ g(x) = \sqrt{3x^2+11}$.

   (b) $g \circ f(x)$
   Solution: $g \circ f(x) = 3x - 1 + 4 = 3x + 3$.

18. (15 points) Let $f(x) = \sqrt{2x-1}$.

   (a) Find the slope of the line joining the points $(5, 3)$ and $(x, f(x))$, where $x \neq 5$.
   Solution: $\frac{\sqrt{2x-1}-3}{x-5}$.

   (b) Then find the limit of the expression in (a) as $x \to 5$. Call this limit $f'(5)$.
   Solution: $\lim_{x\to 5} \frac{\sqrt{2x-1}-3}{x-5} = \lim_{x\to 5} \frac{(2x-1)-9}{(x-5)(\sqrt{2x-1}+3)} = 1/3$.

   (c) Use the information found in (b) to write an equation for the line tangent to the graph of f at the point $(5, 3)$.
   Solution: $y - 3 = (1/3)(x - 5)$, so $y = x/3 + 4/3$.