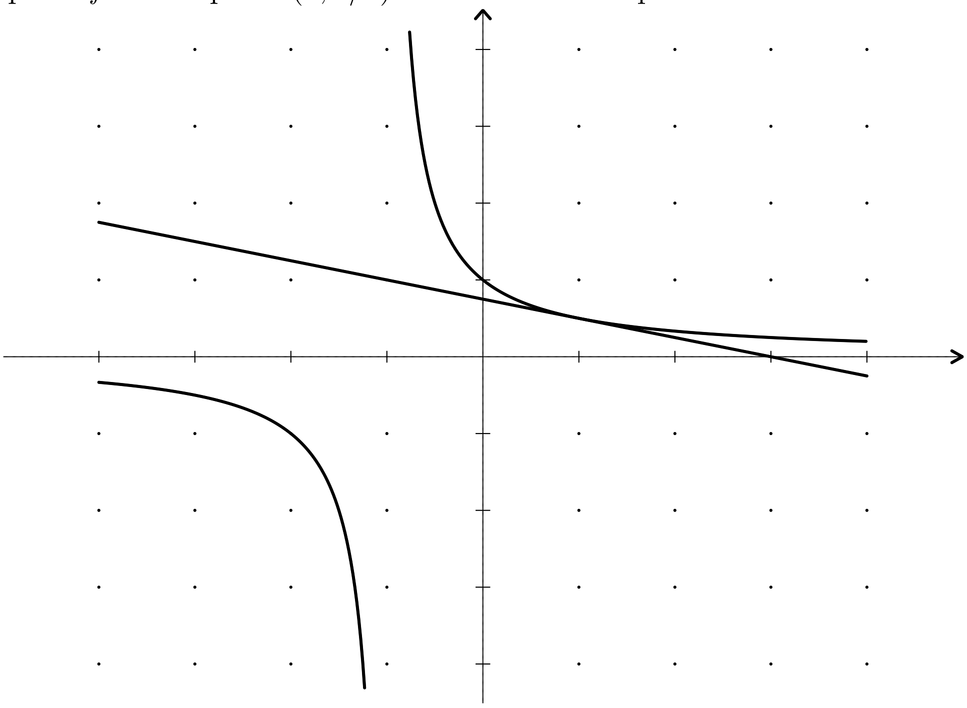
March 3, 2003 **Name** ______________________________

On all the following questions, **show your work.**

1. (20 points) Let $f(x) = \frac{1}{1+x}$. Notice that $f(1) = 1/2$.

   (a) Sketch the graph of f on the grid provided and draw the line tangent to the graph of f at the point $(1, 1/2)$. Estimate the slope of the line.



   (b) Compute $\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$.

Solution: $\lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-h}{2(2+h)} \cdot \frac{1}{h} = \lim_{h\to 0} -\frac{1}{2(2+h)} = -\frac{1}{4}$.

   (c) Describe what the answer to (b) means.

Solution: It means that the slope of the line tangent to the graph of our function f at the point $(1, 1/2)$ is $-1/4$.

2. (20 points) Let $g(x) = \sqrt{x+2}$. Find $g'(a)$ by taking the limit of the difference quotient. In other words, use the definition of derivative.

   Solution:

$$\lim_{h\to 0} \frac{g(a+h)-g(a)}{h} = \lim_{h\to 0} \frac{\sqrt{a+h+2}-\sqrt{a+2}}{h} =$$

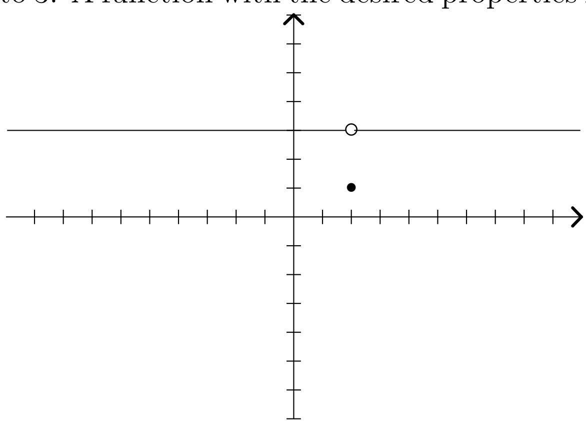
$$\lim_{h\to 0} \frac{\sqrt{a+h+2}-\sqrt{a+2}}{h} \cdot \frac{\sqrt{a+h+2}+\sqrt{a+2}}{\sqrt{a+h+2}+\sqrt{a+2}} =$$

$$\lim_{h\to 0} \frac{a+h+2-(a+2)}{h(\sqrt{a+h+2}+\sqrt{a+h})} =$$

$$\lim_{h\to 0} \frac{1}{\sqrt{a+h+2}+\sqrt{a+h}} = \frac{1}{2\sqrt{2+a}}.$$

3. (20 points) Describe in English what it means to say that "the limit of a function f is 3 as x approaches 2". Sketch the graph of a function which has this property but also satisfies $f(2) = 1$.

   Solution: It means that when x is close to 2, but not equal to 2, $f(x)$ is close (and possibly equal) to 3. A function with the desired properties is given below.



4. (20 points) Let $k(x) = x^2 - x$.

   (a) Using the definition of derivative, find $k'(x)$
   Solution:

$$
\begin{aligned}
k'(x) &= \lim_{h\to 0} \frac{k(x+h) - k(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2\cdot xh + h^2 - x - h - (x^2 - x)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2\cdot xh + h^2 - x - h - x^2 + x)}{h} \\
&= \lim_{h\to 0} \frac{h(2x + h - 1)}{h} = 2x - 1.
\end{aligned}
$$

   (b) Evaluate the function found above at $x = 3$ to find $k'(3)$.
   Solution: Since $k(x) = 2x - 1$ it follows that $k(3) = 2 \cdot 3 - 1 = 5$.

   (c) Use the information above to find an equation for the line tangent to the graph of k at the point $(3, k(3))$.
   Solution: $y - k(3) = 5(x - 3)$, which reduces to $y = 5x - 9$.