Math 1120 **Calculus** **Final Exam, section 6**

December 13, 2011 Name ______________________

The total number of points available on this test is 273.

1. (10 points) Let $f(x) = x^2 - 3x + 5$. Find an equation for the line tangent (in slope-intercept form) to the graph of f at the point $(2, f(2))$.

2. (10 points) The line tangent to the graph of a function f at the point $(3, 5)$ on the graph also goes through the point $(0, 8)$. What is $f'(3)$?

3. (10 points) Suppose $f'(x) = 2x - \ln(x)$ and $f(e) = 3$. Find an equation for the line tangent to the graph of f at the point $(e, 3)$.

4. (10 points) Build the sigh chart for the derivative of $g(x) = e^{x^2-4x}$ and use it to find the intervals over which g is increasing.

5. (15 points) Let $f(x) = 2x^2 - 3x$.

   (a) Find a point $(a, f(a))$ on the graph of f where the tangent line has slope 0.

   (b) Find a point $(a, f(a))$ on the graph of f where the tangent line has slope 1.

   (c) Find a point $(a, f(a))$ on the graph of f where the tangent line has slope 2.

6. (30 points) A manufacture has been selling 1850 television sets a week at \$510 each. A market survey indicates that for each \$10 rebate offered to a buyer, the number of sets sold will increase by 100 per week.

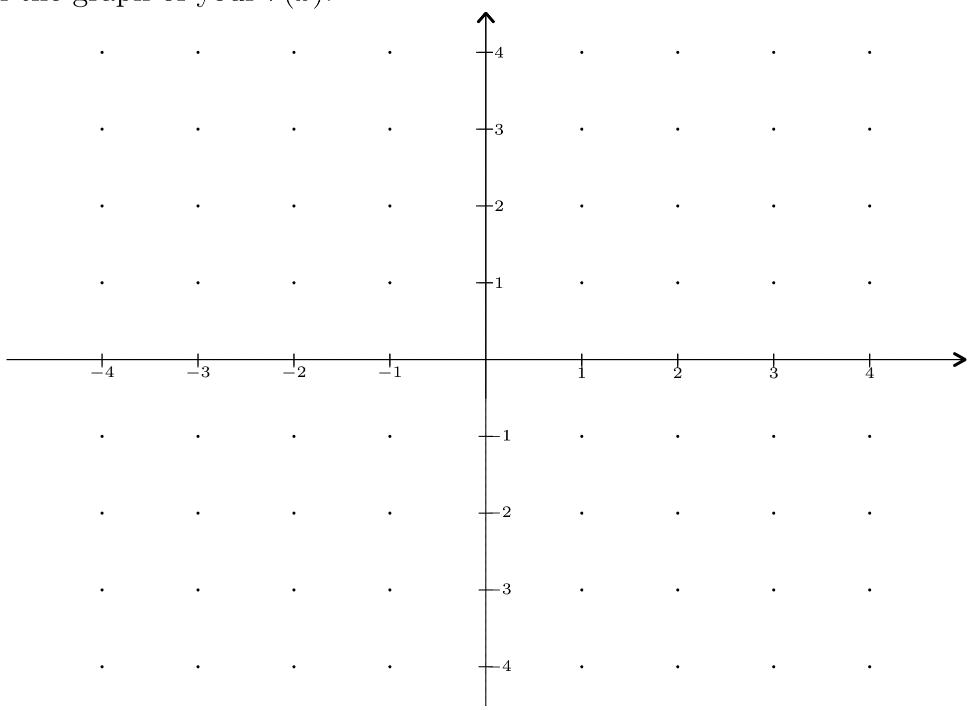
   (a) Find the linear demand function $p(x)$, where x is the number of the television sets sold per week.

   (b) How large rebate should the company offer to a buyer, in order to maximize its **revenue**?

   (c) If the weekly cost function is $157250 + 170x$, how should it set the size of the rebate to maximize its profit?

7. (20 points) Find a rational function $r(x)$ that has exactly two zeros, $x = -2$ and $x = 1$, exactly two vertical asymptotes at $x = -1$ and $x = 3$, and has a horizontal asymptote $y = 2$.

   (a) Sketch the graph of your $r(x)$.



   (b) Find a symbolic representation of r.

   (c) Find the derivative of your function r and build the sign chart for $r'(x)$. Is there an interval over which r is increasing? If so, find it.

8. (12 points)

   (a) Find the rate of change of $f(x) = x^2 \ln(2x + 1)$ when $x = 1$.

   (b) Find the slope of the line tangent to f at the point $(2, 4 \ln 5)$.

9. (12 points) If we add a number to three times a second number, we get the answer 48. What is the largest possible product the two numbers can have?

10. (12 points) An investment of \$1000 that is compound continuously takes exactly 11 years to double in value.

    (a) How long does it take to triple?

    (b) How long does it take to grow \$1000 into \$8000?

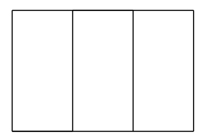
    (c) Find a function that describes the amount in the account after t years?

    (d) Find the value of your function at $t = 33$ years.

11. (12 points) If $h = g \circ f$ where $f(x) = x - 1/x,\ g(x) = x^2 + x + 2$. Find $h'(x)$. Then find $h'(1)$ and use that number to find an equation for the line tangent to the graph of $h(x)$ at the point $(1, 2)$.

12. (15 points) Let $f(x) = x^2 e^{3x}$. Find the interval(s) where f is concave upward.

13. (15 points) A rancher wants to fence in an area of 12 square miles in a rectangular field and then divide it into three pastures with two fences parallel to one side as shown below. What is the shortest length of fence that the rancher can use?



14. (10 points) The line $2x + 3y = 13$ is tangent to the graph of a function $g(x)$ at the point $(2, 3)$. What is $g'(2)$?

15. (10 points) There is exactly one function $f(x)$ such that $f'(x) = e^x - x^e$ and $f(0) = 2$. Find the value of this function at $x = 1$. Leave your answer as a decimal accurate to the nearest tenth.

16. (70 points) Evaluate each of the following integrals using the Fundamental Theorem of Calculus (ie, antidifferentiate, then measure the growth of an antiderivative over the interval).

    (a) Evaluate $\displaystyle\int_0^e \frac{2x}{x^2+1}\,dx$

    (b) Evaluate $\displaystyle\int_{-1}^{1} (x+2)^5 x\,dx$. Leave your answer as a decimal accurate to the nearest hundredth.

    (c) Evaluate $\displaystyle\int_0^2 x^2 - \sqrt{x}\,dx$

    (d) Evaluate $\displaystyle\int_{-1}^{1} \frac{d}{dx}\left(5x^4 - 3x^3 + 7x\right)\,dx$.

(e) Evaluate $\displaystyle\int_0^4 \frac{x^3+8}{x+2}\,dx$

(f) Evaluate $\displaystyle\int_1^3 x^3 \cdot (x^4-2)^2\,dx$

(g) Evaluate $\displaystyle\int_0^4 2e^{2x}\,dx$