May 11, 2011 **Name** _____

The total number of points available is 305. Throughout the free response part of this test, **show your work.** Each of the first 20 problems is worth 10 points.

1. Let $f(x) = x^4 - 2x + 4$. What is $f'(1)$?

2. Find an equation for the line tangent to the graph of $f(x) = 3x^3 - 2x + 4$ at the point $(2, f(2))$?

3. Consider the function $f(x) = (e^{2x} + 1)^3$. What is the slope of line tangent to the graph of f at the point $(1, f(1))$?

4. Suppose the line $3x + 4y = 11$ is tangent to the graph of $h(x)$ at the point $(1, 2)$. What is $h'(1)$?

5. What is $\lim_{x\to\infty} \dfrac{(3x+2)(4x-1)}{(x-2)(2x-3)}$?

6. What is the exact value of $|2\pi - 7| + |8 - 2\pi| + \pi$? Leave your answer in terms of π. No credit for a decimal approximation.

7. What is $\lim_{x\to 2} \dfrac{x^2-4}{x^3-8}$?

8. Find a function f that satisfies (a) $f'(x) = 3x^2 - 2x$ and (b) $f(2) = 3$.

9. Let $H(x) = \ln(4x^2 + 12x + 10) - 2x$. Find all critical points of H.

10. Let $g(x) = 2x^3 - 7x^2 + 4x - 10$. Find the intervals over which g is decreasing?

11. Let $k(x) = 2x^4 - 14x^3 + 30x^2 + 10x$. Over which intervals is k is concave upwards?

12. What is the value of $\int_2^4 \frac{d}{dx}(3x - 5)^4 \, dx$

13. What is the area of the region R bounded above by $y = 2x + 1$, below by $y = x - 7$, on the left by $x = 2$ and on the right by $x = 4$?

14. Find a value of b for which $\displaystyle\int_b^{2b} \frac{1}{x} + 1\, dx = \ln(2) + 6$.

15. What is the absolute maximum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 5$ on the interval $-2 \leq x \leq 3$?

16. Find all the zeros of the polynomial $p(x) = (x-1)^3(x+2)^2 - 4(x-1)^2(x+2)$.

17. Use calculus to find $\int e^{2x}(e^{2x} + 1)^4\, dx$.

18. Use calculus to find $\int \frac{2x}{x^2+1}\, dx$.

19. Use calculus to compute $\int_1^3 x^2 - x - \frac{1}{x} + 1\, dx$.

20. Given that the graph of f passes through the point $(1, 5)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$, what is $f(4)$?

21. (15 points) Rachel learns typing in a 14 week class. The number of words per minute Rachel can type after t weeks is given by

$$F(t) = 120 - 40e^{-0.4t}.$$

(a) How many more words per minute can Rachel type after the third week than she can type after the second week? (b) What is $F'(2.5)$? (c) How are these numbers related?

22. (20 points) Find the area of the region caught between the functions $f(x) = 5 - x^2$ and $g(x) = 2x - 3$. Show how you used the Fundamental Theorem by measuring the growth of an antiderivative over an interval. Your work must make clear what interval you used.

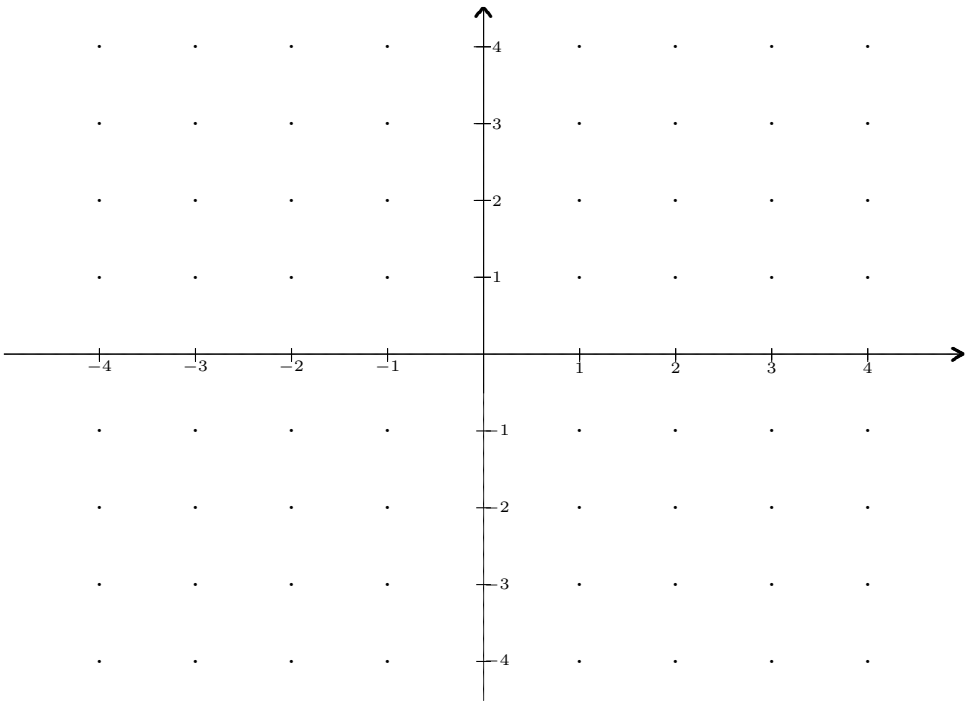
23. (30 points) Let $h(x) = \frac{x(2x+11)(2x+7)}{(x-1)^2(3x-12)}$.

(a) Find the asymptotes of h.

(b) Find the zeros of h.

(c) Build the sign chart for $h(x)$.

(d) Sketch the graph of $h(x)$ USING the information in (a) and (b).



24. (20 points) Let $H(x) = \sqrt{(3x+1)^{12}+3}$.

    (a) Find three functions f, g and h satisfying $f(g(h(x))) = f \circ g \circ h(x) = H(x)$.

    (b) Compute the derivative of each of the three component functions f, g, h.

    (c) Apply the chain rule twice to find $H'(x)$.

25. (20 points) The quadrilateral T with vertices $A = (0,0)$, $B = (0,6)$, $C = (8,10)$ and $D = (8,0)$ is a trapezoid since the two sides AB and CD are both vertical. It is not hard to see that the area of T is 64 square units.

    (a) Find an equation for the line passing through the points B and C. Let $f(x)$ be the function whose graph is this line.

    (b) Use calculus, showing all your work, to verify that the area of the region T bounded above by the graph of f, below by the x-axis, and on the sides by $x = 0$ and $x = 8$ is 64.