Name ____________________

There are 210 points available on this test.

1. (10 points) The line tangent to the graph of a function f at the point $(2, 9)$ on the graph also goes through the point $(0, 7)$. What is $f'(2)$?

   Solution: The slope of the line through $(2, 9)$ and $(0, 7)$ is 1, so $f'(2) = 1$.

2. (10 points) Find an equation for the line tangent to the graph of $f(x) = x^2 - 3x$ at the point (2,-2)?

   Solution: The derivative is $f'(x) = 2x - 3$ whose value of at $x = 2$ is $f'(2) = 4 - 4 = 1$. Thus the line is $y - (-2) = 1(x - 2)$, which, in slope intercept form is $y = x - 4$.

3. (10 points) Find an equation for the line tangent to the graph of $f(x) = \ln(2x + 1)$ at the point (0, 0)?

   Solution: The derivative is $f'(x) = \frac{2}{2x+1}$ whose value of at $x = 0$ is $f'(0) = 2$. Thus the line is $y - 0 = 2(x - 0)$, which, in slope intercept form is $y = 2x$.

4. (10 points) Find an equation for the line tangent to the graph of $y = e^{(2x-1)}$ at the point on the graph where $x = 2$?

   Solution: The derivative is $f'(x) = 2e^{2x-1}$ whose value of at $x = 2$ is $f'(2) = 2e^3$. Thus the line is $y - e^3 = 2e^3(x - 2)$, which, in slope intercept form is $y = 2e^3 x - 3e^3$.

5. (10 points) Find the rate of change of $f(t) = e^{3t} \cdot \ln(t)$ when $t = 1$.

   Solution: Use the product rule to get $f'(t) = 3e^{3t} \cdot \ln(t) + (1/t) \cdot e^{3t}$ whose value at $t = 1$ is $f'(1) = 3e^3 \cdot \ln(1) + (1/1) \cdot e^3 = e^3$, since $\ln 1 = 0$.

6. (10 points) Let $h(x) = \frac{\sqrt{(x-4)(x-2)(2x+7)}}{x^2-100}$. Write the domain of h in interval notation.

   Solution: There are two things to guard against, division by zero and having a negative sign in the radical. Division by zero happens when either $x = 10$ or $x = -10$. So we need the sign chart for $(x-4)(x-2)(2x+7)$, which clearly has zeros at $x = 2, x = 4$, and $x = -7/2$. The sign chart reveals that the product is positive over $(-7/2, 2)$ and $(4, \infty)$. Therefore, the answer we seek is $[-7/2, 2] \cup [4, 10) \cup (10, \infty)$.

7. (20 points) Let $h(x) = \ln(x^2 + 4x + 5)$.

   (a) What is the domain of h. Recall that $\ln(x)$ is defined only if $x > 0$.

   Solution: We need to find all values of x for which x^2+4x+5 is positive. Compute the discriminant $D = b^2 - 4ac = 16 - 20 < 0$ to see that our parabola lies completely above the x-axis. Therefore the domain of h is the whole real line.

   (b) Build the sign chart for $h'(x)$.

   Solution: Since $h'(x) = \frac{2x+4}{x^2+4x+5}$, it follows that the sign chart of h' is the same as that of $2x + 4$, which requires straightforward calculation.

   (c) Discuss the local max and min of h.

   Solution: From the part (b), we can see that h is decreasing up to $x = -2$ and increasing after that, so h must have a minimum at $x = -2$.

8. (15 points) A radioactive substance has a half-life of 22 years. Find an expression for the amount of the substance at time t if 20 grams were present initially.

   Solution: $Q(t) = Q_0 e^{-kt}$. Since $Q_0 = 20$ and the half-life is 22 years, it follows that $10 = 20e^{-22k}$, which can be solved to give $k \approx 0.0315$. Thus $Q(t) = 20e^{-0.0315t}$.

9. (10 points) If $h = g \circ f$ and $f(1) = 3, g'(3) = 7, f'(1) = -2$ find $h'(1)$.

   Solution: $h'(1) = g'(f(1)) \cdot f'(1) = g'(3) \cdot f'(1) = -14$.

10. (15 points) Let $f(x) = x^4 + 2x^3 - 12x^2 + x - 5$.

    (a) Find the interval(s) where f is concave upward.

    Solution: $f'(x) = 4x^3 + 6x^2 - 24x + 1$ and $f''(x) = 12x^2 + 12x - 24$, which has two zeros, $x = 1$ and $x = -2$. So f'' is positive over the intervals $(-\infty, -2)$ and $(1, \infty)$.

    (b) Find the inflection points of f, if there are any.

    Solution: There are two inflection points, $(1, -14)$ and $(-2, -55)$

11. (15 points) Find the area of the region R bounded above by the graph of $f(x) = x^2 - 3x + 11$, below by the x-axis, and on the sides by the vertical lines $x = 0$ and $x = 2$.

    Solution: The integral is $\int_0^2 x^2 - 3x + 11 \; dx = (x^3/3 - 3x^2/2 + 11x|_0^2) = (8/3 - 6 + 22) - 0 = 56/3$.

12. (15 points) Find the area of the region R caught between the graph of $f(x) = x^2 - 3x + 2$ and $g(x) = -x + 5$.

    Solution: First note that the two functions agree at $x = -1$ and $x = 3$. You can make this discovery either by factoring the difference function or using the graphing calculator. The area then is the value of the integral $\int_{-1}^3 g(x) - f(x) \; dx = \int_{-1}^3 -x^2 + 2x + 3 \; dx = -x^3/3 + x^2 + 3x|_{-1}^3 = 9 + 1/3 - 4 = 32/3$.

13. (40 points)

(a) Evaluate $\int x^3 - x^{-2} + x^{-1}\ dx$
Solution: $\int x^3 - x^{-2} + x^{-1}\ dx = x^4/4 + x^{-1} + \ln(x) + C$.

(b) Evaluate $\int_1^3 \frac{x^3-2x^2+x}{x}\ dx$
Solution: Its just $x^3/3 - x^2 + x|_1^3 = (9-9+3) - (1/3-1+1) = 8/3$.

(c) Evaluate $\int_0^7 \frac{d(x-5)^9}{dx}\ dx$
Solution: Its just $(x-5)^9|_0^7 = 2^9 - (-5)^9 = 2^9 + 5^9 = 1953637$.

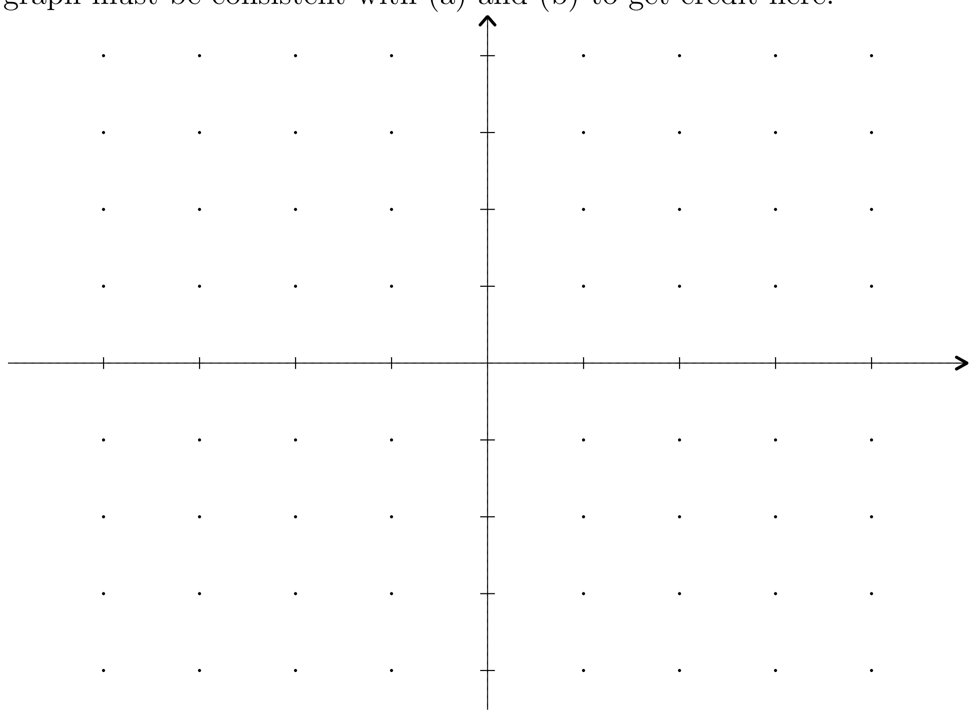
(d) Evaluate $\int_0^4 \frac{3x^2}{x^3+5}\ dx$
Solution: $\int_0^4 \frac{3x^2}{x^3+5}\ dx = \ln(x^3+5)|_0^4 = \ln(69) - \ln 5 \approx 2.625$.

14. (20 points)

(a) Find the sign chart for the function $g(x) = \frac{(2x-3)(3x+1)}{(x-4)(x+2)}$.
Solution: The branch points are the zeros of the numerator and of the denominator, namely $x = 3/2, x = -1/3$, $x = -2$ and $x = 4$. Build the sign chart to reveal that g is positive on the first, third and fifth of the five intervals.

(b) Find all the asymptotes of g.
Solution: The vertical asymptotes are clearly $x = -2$ and $x = 4$. The horizontal asymptote is the ratio of the coefficients of the x^2 terms, namely 6 and 1. So $y = 6$ is the horizontal asymptote.

(c) Use the information in (a) and (b) to sketch the graph of g. Note: the graph must be consistent with (a) and (b) to get credit here.



Solution: I'm not able to draw the graph at this time.