May 6, 2008 Name _____

The total number of points available on this test is 239. **Show all your work.** If you use decimal notation, carry out the arithmetic to three places to the right of the decimal and round off to two places.

1. (12 points) The line tangent to the graph of a function f at the point $(2, 5)$ on the graph also goes through the point $(0, 11)$. What is $f'(2)$?

2. (20 points) Let $f(x) = \sqrt{x-2}$.

   (a) Compute the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

   (b) Compute $lim_{h\to 0}\dfrac{f(x+h) - f(x)}{h}$.

   (c) Use the information in part (b) to find $f'(3)$.

   (d) Use the information above to find an equation for the line tangent to f at the point $(3, f(3))$.

3. (12 points) Suppose $f'(x) = x - \ln(x)$ and $f(e) = 3$. Find an equation for the line tangent to the graph of f at the point $(e, 3)$.

4. (15 points) Let $f(x) = \sqrt{9 - |x-5|}$. Use the test interval method to find the domain of f.

5. (12 points)

   (a) Find the rate of change of $f(x) = x^2 \ln(2x+1)$ when $x = 1$.

   (b) Find the slope of the line tangent to f is the point $(2, 4\ln 5)$.

6. (12 points) A radioactive substance has a half-life of 37 years. Find an expression for the amount of the substance at time t if 20 grams were present initially.

7. (12 points) If $h = g \circ f$ and $f(1) = 2, g'(2) = 5, f'(1) = -3$ find $h'(1)$.

8. (15 points) Let $f(x) = e^{2x}/x^2$. Find the interval(s) where f is concave upward.

9. (15 points) Find the area of the region R bounded above by the graph of $f(x) = -(x+1)(x-3)$, below by the x-axis, and on the sides by the vertical lines $x = 0$ and $x = 3$.

10. (15 points) Find the area of the region R caught between the graph of $f(x) = x^2 - 3x + 2$ and $g(x) = -x + 5$.

11. (15 points) Find all asymptotes of the rational function $r(x) = \dfrac{(x^3 - 64)(x^2 - 9)}{3(x^2 - 16)(x + 3)(x^2)}$.

12. (18 points) If a ball is thrown vertically upward from the roof of 128 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 128 + 64t - 16t^2$. Be sure to show your work and explain each step in English.

    (a) What is the height the ball at time $t = 1$?

    (b) What is the velocity of the ball at the time it reaches its maximum height?

    (c) What is the maximum height the ball reaches?

    (d) After how many seconds is the ball exactly 160 feet above the ground?

    (e) How fast is the ball going the first time it reaches the height 160?

    (f) How fast is the ball going the second time it reaches the height 160?

13. (10 points) Evaluate $\displaystyle\int x^2 - \sqrt{x} - \frac{1}{x}\, dx$

14. (10 points) Evaluate $\displaystyle\int x^2\sqrt{x^3 + 4}\, dx$

15. (36 points) Evaluate each of the following integrals using the Fundamental Theorem of Calculus (ie, antidifferentiate, then measure the growth of an antiderivative over the interval).

    (a) Evaluate $\displaystyle\int_0^4 \frac{x^3 + 8}{x + 2}\, dx$

    (b) Evaluate $\displaystyle\int_1^3 x^3 \cdot (x^4 - 2)^2\, dx$

    (c) Evaluate $\displaystyle\int_0^4 2xe^{x^2}\, dx$