December 12, 2006 **Name** ____________________

The total number of points available is 192. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = x^2 e^x$.
   (a) Find the values of x at which the line tangent to the graph of f is horizontal.
   (b) Find the intervals over which the function f is increasing.

2. (30 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 1)(x - 3)^2(x - 5)(3x + 13).$$

   What this says is that u has already been differentiated and the function given is $u'(x)$. Recall that an important theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.
   (a) Find the critical points of $u(x)$.
   (b) Use the Test Interval Technique to find the intervals over which $u'(x)$ is positive.
   (c) Use the information in part (b) to find the intervals over which $u(x)$ is increasing.

3. (25 points) Consider the function $f(x) = 36x^3 - 21x^2 + 12x - 7$.
   (a) Find an antiderivative of $f(x)$.
   (b) Find an antiderivative of $f(x)$ whose value at $x = 3$ is 2.
   (c) Compute $\int_0^1 f(x)$.

4. (20 points) Consider the function $f(x) = (x^2 - 4)^{2/3}$.
   (a) Find $f'(x)$ and $f''(x)$. Discuss their domains.
   (b) Find the critical points of f and identify them as stationary or singular.
   (c) Use the sign chart for f' to decide, for each critical point, whether a local maximum, local minimum, or neither occurs at that critical point.

5. (20 points) Let $f(x) = \frac{4}{x} - 2e^x$.
   (a) Find an antiderivative of $f(x)$.
   (b) Find an antiderivative of $f(x)$ with a value of 4 at the point $x = 1$.
   (c) Compute $\int_1^4 f(x)$.

6. (10 points) What is the value of $\int_0^4 2x\sqrt{x^2+1}\, dx$?

7. (15 points) Compound Interest.
   (a) Consider the equation $1000(1+0.01)^{12t} = 3000$. Find the value of t and interpret your answer in the language of compound interest. In other words, write a sentence about what the value of t is.
   (b) Consider the equation $P(1+0.03)^{4\cdot 10} = 3000$. Solve for P and interpret your answer in the language of compound interest.
   (c) Consider the equation $1000e^{8r} = 2000$. Solve for r and interpret your answer in the language of compound interest.

8. (10 points) Find the value of $\int_0^{\sqrt{\ln 6}} \frac{de^{x^2}}{dx}$. Hint: This problem is much easier than it looks.

9. (20 points) A manufacture has been selling 1300 television sets a week at \$450 each. A market survey indicates that for each \$20 rebate offered to a buyer, the number of sets sold will increase by 270 per week. In other words, if they drop the price by \$20, they sell 270 more sets, etc.
   (a) Find the demand function $p(x)$, where x is the number of the television sets sold per week, and $p(x)$ is measured in dollars.
   (b) How large rebate should the company offer to a buyer, in order to maximize its revenue?
   (c) If the weekly cost function is $97500 + 150x$, how should it set the size of the rebate to maximize its profit?

10. (12 points) The population of the world in 1987 was 5 billion and the relative growth rate was estimated at 1.4 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 1995.

11. (15 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 135e^{-0.01t}$$

where $m(t)$ is measured in grams.

(a) Find the mass at time $t = 0$.

(b) How much of the mass remains after 10 years?

(c) What is the half-life of the material?