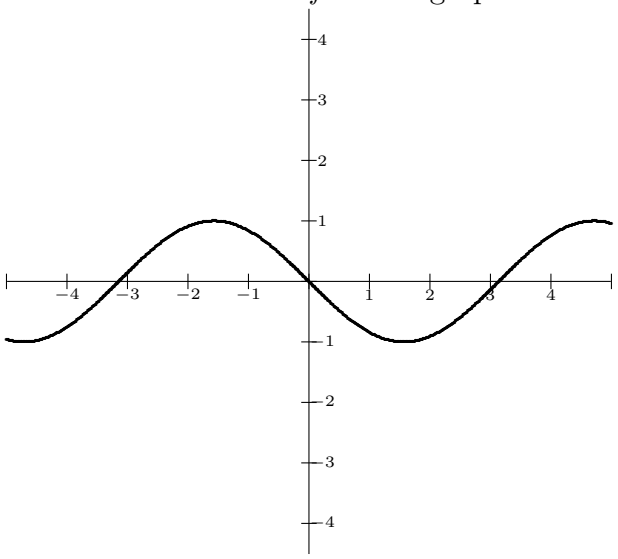
December 13, 2004 **Name** _____

Each of the 12 multiple choice problems counts 8 points. The other problems count as marked. The total value of these problems is 278 points. On the free response part of this test, **show your work.** For some questions, the answer alone without any supporting mathematics will not be worth any points. Note that some multiple choice problems have more than one correct answer. Circle *all* the options that apply.

1. Consider the function f whose graph is shown below.



(a) Which of the following could be a tangent line for (the graph of) f?

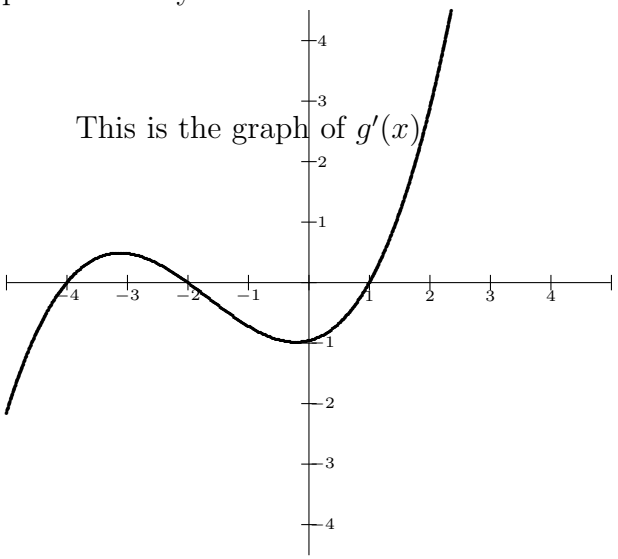
(A) $y - 1 = 1(x - 1)$ **(B)** $y = 2(x - 1)$ **(C)** $y = -x$

(D) $y = -2(x - 2)$ **(E)** $y - 1 = -x$

(b) Again referring to the f in problem 1, over which interval is f decreasing? Circle all the options that apply.

(A) $[-4, -2]$ **(B)** $[-1, 1]$ **(C)** $[0, 3]$ **(D)** $[2, 3]$ **(E)** $[3, 4]$

2. A function g has a *derivative* whose graph is shown below. Recall that some problems may have several correct answers. Circle them all.



   (a) At which of the following points is it true that g' is increasing? Circle all that apply.

   (A) -4 **(B)** -3 **(C)** -1 **(D)** 1 **(E)** 2

   (b) At which of the following points is it true that g is increasing?

   (A) -4 **(B)** -3 **(C)** -1 **(D)** 1 **(E)** 2

   (c) Again referring to the graph of g' above, at which of the points could $g''(x)$ have the value zero?

   (A) -4 **(B)** -3 **(C)** -1 **(D)** 0 **(E)** 1

   (d) Again referring to the graph of g' above, at which of the points could $g'''(x)$ have the value zero?

   (A) -4 **(B)** -3 **(C)** -1 **(D)** 1 **(E)** 2

3. An amount of \$2000 is invested at $r\%$ interest compounded continuously. After four years, the account has grown to \$2800. Assuming that it continues to grow at this rate for 16 more years, how much will be in the account?

   (A) \$8976.47 **(B)** \$9874.23 **(C)** \$10001.99

   (D) \$10756.48 **(E)** \$2004.35

4. For each of the next five problems, refer to the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Which of the following is an equation for the line tangent to the graph of f at the point $(2, f(2))$?

(A) $y - 6 = 4(x - 2)$ **(B)** $y - 4 = 2(x - 6)$ **(C)** $y - 2 = 4(x - 6)$
(D) $y - 6 = 2(x - 4)$ **(E)** $y - 2 = 6(x - 4)$

(b) What is the value of $f(g(f(g(3))))$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

(c) What is the value of $f'(g(f(g'(1))))$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

(d) Let $h(x) = \frac{d}{dx} f \circ g$. What is the value of $h(3)$?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

(e) Let $k(x) = g \circ f$. What is the value of $k'(2)$?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 16

(f) What is the slope of the line joining the points $(1, f(1))$ and $(2, g(2))$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

5. (40 points) This question is about building more complicated functions from simpler ones. Let $f(x) = x^2$, $g(x) = \sqrt{x}$, $h(x) = x + 1$, $k(x) = 1/x$ and $l(x) = x - 2$. For each function given below, show how it is possible to combine some of the simpler functions above to obtain the given one. For example, if $U(x) = \sqrt{x^2 - 2}$ was given, you could write $U(x) = g \circ l \circ f(x)$, and if $V(x) = ((x + 1)/x)^2$, you could write $V(x) = f \circ (h \cdot k)(x)$.

(a) $H(x) = \left(\frac{1}{x-2}\right)^2 + 1$

(b) $G(x) = \left(\frac{1}{x-2} + 1\right)^2$

(c) $L(x) = \frac{x+1}{x-2} - 2$

(d) $K(x) = \frac{1}{(x+1)^2-2}$

(e) $N(x) = \sqrt{(x-2)^2+1}$

6. (30 points) Let $R(x)$ be the rational function defined by

$$R(x) = \frac{(x+3)(x-4)(2x-7)}{(x+1)^2(x-1)}.$$

   (a) At which of the following points is R positive? Circle all the apply.

   (A) -5 **(B)** -3 **(C)** -2 **(D)** 0 **(E)** 3

   (b) At which of the following points does R change signs? Circle all the apply.

   (A) -3 **(B)** -1 **(C)** 1 **(D)** $7/2$ **(E)** 4

   (c) What is $\lim_{x\to\infty} R(x)$?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** This limit does not exist

7. (30 points) Suppose we know that the function f has been differentiated and that $f'(x) = 2x(x^2-3)^4$. Also, the point $(2, 1/5)$ belongs to the graph of f.

   (a) Find an equation for the line tangent to the graph of f at the point $(2, 1/5)$.

   (b) Find $f(1)$. Hint: f is an antiderivative of f'.

   (c) Find the area of the region R bounded above by the graph of $f'(x)$, below, by the x-axis and on the sides by the lines $x = 0$ and $x = 1$.

8. (42 points)

   (a) $\int 4x - 5 \, dx$

   (b) $\int 9x^2 - 4x - 1/x \, dx$

   (c) $\int \dfrac{x^3 + 2x^2 - x}{x} \, dx$

   (d) $\int \dfrac{2x+3}{x^2+3x-3} \, dx$

   (e) $\int 6x^5(x^6+3)^7 \, dx$

   (f) $\int x^2 e^{x^3} \, dx$

9. (10 points) Find an equation for the line tangent to the graph of $f(x) = x\ln(x) - x$ at the point $(1, f(1))$.

10. (30 points) Let $g(x) = (x-1)(x+1)(x-3)$ and let $f(x) = 2(x-1)(x-3)$.

(a) Find the two values of x for which $f(x) = g(x)$. In other words, where do the graphs intersect. Hint: solve $g(x) - f(x) = 0$.

(b) Set up an integral whose value is the area of the bounded region R caught between the two graphs.

(c) Evaluate this integral to find the area of R.