

May 7, 2004

Name \_\_\_\_\_

The first six problems count 7 points each (total 42 points) and rest count as marked.

There are 207 points available. Good luck.

1. Consider the function  $f$  defined by:

$$f(x) = \begin{cases} x^2 - 3 & \text{if } x < 0 \\ 5x - 3 & \text{if } x \geq 0 \end{cases}$$

Find the slope of the line which goes through the points  $(-2, f(-2))$  and  $(3, f(3))$ .

- (A)  $7/5$    (B)  $11/5$    (C)  $17/5$    (D)  $5$    (E)  $7$
2. The distance between the point  $(6.5, 8.5)$  and the midpoint of the segment joining the points  $(1, 5)$  and  $(2, 7)$  is
- (A)  $\sqrt{22}$    (B)  $\sqrt{23}$    (C)  $5\sqrt{5}/2$    (D)  $\sqrt{26}$    (E)  $6$
3. Let  $f(x) = 2x + 3$  and  $g(x) = 3x - 6$ . Which of the following does not belong to the domain of  $f \circ g$ ?
- (A)  $1$    (B)  $2$    (C)  $3$    (D)  $4$
- (E) The domain of  $f \circ g$  is the set of all real numbers.
4. The line tangent to the graph of a function  $f$  at the point  $(2, 5)$  on the graph also goes through the point  $(0, 7)$ . What is  $f'(2)$ ?
- (A)  $-2$    (B)  $-1$    (C)  $0$    (D)  $1$    (E)  $2$
5. What is the slope of the tangent line to the graph of  $f(x) = x^{-1}$  at the point  $(3, 1/3)$ ?
- (A)  $-1$    (B)  $-1/2$    (C)  $-1/3$    (D)  $-1/9$    (E)  $1/3$
6. The line tangent to the graph of the function  $f(x)$  at the point  $(2, 5)$  is  $2y - 3x = 4$ . What is  $f'(2)$ ?
- (A)  $0$    (B)  $2/3$    (C)  $3/2$    (D)  $-2/3$    (E)  $-3/2$
7. (15 points) Let  $f(x) = \sqrt{2x - 1}$ .
- (a) Construct  $\frac{f(5+h) - f(5)}{h}$

- (b) Simplify and take the limit of the expression in (a) as  $h$  approaches 0 to find  $f'(5)$ .
- (c) Use the information found in (b) to find an equation for the line tangent to the graph of  $f$  at the point  $(5, 3)$ .
8. (20 points) Find the interval(s) where  $f(x) = (x - 4)(x^2 - 1)(x + 3)$  is positive.
9. (15 points) Let  $f(x) = 4x^3 + 6x^2 - 24x + 1$ .
- (a) Find the interval(s) where  $f$  is decreasing.
- (b) Find all inflection points of  $f$ .
10. (20 points) A ball is thrown upwards from the top of a building that is 200 feet tall. The position of the ball at time  $t$  is given by  $s(t) = -16t^2 + 36t + 200$ , where  $s(t)$  is measured in feet and  $t$  is measured in seconds.
- (a) What is the position of the ball at time  $t = 0$ ?
- (b) What is the velocity of the ball at time  $t = 0$ ?
- (c) What is the acceleration of the ball at time  $t = 0$ ?
- (d) What is the velocity of the ball at time  $t = 1$ ?
- (e) How many seconds elapse before the ball hits the ground?
- (f) What is the speed of the ball when it hits the ground?
- (g) What is the acceleration of the ball at the time it hits the ground?

11. (15 points) Find an equation for the line tangent to the graph of  $f(x) = x \ln(x) - x$  at the point  $(1, f(1))$ .

12. (20 points) Find the absolute maximum value of the function

$$f(x) = x^3 - 6x^2 + 9x - 5$$

over the interval  $[0, 4]$ .

13. (60 points) Find the following antiderivatives and definite integrals.

(a)  $\int 6x^3 - 5x - 1 dx$

(b)  $\int 4x^{\frac{5}{2}} + x^{-\frac{1}{2}} dx$

(c)  $\int \frac{3x^4 + 2x^2 - 1}{x^2} dx$

(d)  $\int \frac{2x + 1}{x^2 + x - 3} dx$

(e)  $\int 5x^4(x^5 + 2)^3 dx$

(f)  $\int_0^1 2x^2 - 3x dx$

(g)  $\int_0^2 xe^{x^2} dx$

(h) Find the derivative of  $g(x) = x \ln x$ . Evaluate  $\int_1^e \ln x dx$ .