Math 1120 **Calculus** **Final Exam**

May 7, 2003. Your name ____________________

As usual, **show all your work**. If you used a calculator, explain in detail how you used it to solve the problem. There are 237 points available on this test.

1. (42 points) Find the following antiderivatives.

   (a) $\int 2x - 3dx$

   Solution: $x^2 - 3x + c$.

   (b) $\int 6x^2 - 4x - 1dx$

   Solution: $2 \cdot x^3 - 2 \cdot x^2 - x + c$.

   (c) $\int \frac{x^3 + 2x - 1}{x}dx$

   Solution: $\int x^2 + 2 - 1/xdx = x^3/3 + 2x - \ln|x| + c$.

   (d) $\int \frac{4x+1}{2x^2 + x - 3}dx$

   Solution: By substitution, $(u = 2x^2 + x - 3)$, $\int \frac{4x+1}{2x^2+x-3}dx = \ln|2x^2 + x - 3| + c$.

   (e) $\int 5x^4(x^5 + 3)^7dx$

   Solution: By substitution with $u = x^5+3$, $\int 5x^4(x^5+3)^7dx = \frac{(x^5+3)^8}{8}+C$.

   (f) $\int 3x^2e^{x^3}dx$

   Solution: By substitution with $u = x^3, du = 3x^2$, $\int e^u du = e^u + C = e^{x^3} + C$.

2. (30 points) Note that $g(x) = (x-1)(x-3)$ has two zeros in the interval $[0, 4]$.

   (a) Find the area of the 'triangular' region bounded by (i) the x-axis, (ii) the y-axis, and (iii) the graph of $g(x)$.
   Solution:

   (b) Compute $\int_0^4 g(x)\,dx$.
   Solution:

   (c) Find the area of the region caught between the graph of $g(x)$ and the x-axis over the interval from $x = 0$ to $x = 4$. Explain why this is different from the number found in part b.
   Solution:

3. (20 points) Find a function $G(x)$ whose derivative is $3x^2 - 7x + 3$ and for which $G(2) = -3$.

   Solution: $G(x) = x^3 - 7x^2/2 + 3x + C$ for some constant C. But since $G(2) = -3 = 2^3 - 28/2 + 6 + C$, it follows that $C = 8 - 14 + 6 = 0$ and $G(x) = x^3 - 7x^2/2 + 3x$.

4. (40 points) Let $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x-1}$, and $h(x) = 2x - 3$. Find each of the functions.

   (a) $\frac{d}{dx}(f \circ g(x))$
   Solution: By the chain rule,

   (b) $h'(g'(x))$
   Solution:

   (c) $\frac{d}{dx}(h \circ h(x))$
   Solution:

   (d) $\frac{d}{dx}(h(x) \cdot (g(x))^2)$
   Solution:

   (e) $\frac{d}{dx}(h(x) \div g(x))$
   Solution:

5. (20 points) Let $g(x)$ be defined as follows: Let

$$g(x) = \begin{cases} e^{2x} & \text{if } x \leq 1 \\ \ln(x-1) & \text{if } x > 1 \end{cases}$$

   (a) Compute the derivative of $g(x)$.
   Solution:

   (b) What is the slope of the line tangent to the graph of $g(x)$ at the point $(0, 1)$.
   Solution:

   (c) What is the slope of the line tangent to the graph of $g(x)$ at the point $(3, \ln(2))$.
   Solution:

   (d) Find an equation for the line tangent to the graph of $g(x)$ at the point $(3, \ln(2))$.
   Solution: First note that $g'(3) = 1/2$ and that $g(3) = \ln(2)$ so the line is $y - \ln(2) = \frac{1}{2}(x-3)$, which simplifies to $y = (x-3)/2 + \ln(2) = x/2 - 3/2 + \ln(2)$.

6. (40 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 4)(x - 1)^2(x + 3)(x + 5).$$

Recall that a major theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.

(a) Find the critical points of $u(x)$.
Solution: $x = -2, x = 2, x = 1, x = -3$ and $x = -5$.

(b) Use the Test Interval Technique to find the intervals over which $u(x)$ is increasing.
Solution: u is increasing over $(-\infty, -5), (-3, -2)$ and over $(2, \infty)$.

7. (25 points) Consider the function $h(x) = \sqrt{2x^3 - 3x^2 - 36x + 500}$ defined over the interval $[-5, 5]$.

   (a) Find $h'(x)$.

   Solution: Differentiate to get $h'(x) = 6(x-3)(x+2)(2x^3 - 3x^2 - 36x + 500)^{-1/2}$

   (b) Find the critical points of $h(x)$.

   Solution: The only critical points are $x = 3$ and $x = -2$.

   (c) Find the absolute maximum and absolute minimum of the $h(x)$ over its domain.

   Solution: Checking endpoints, we have $h(-5) = \sqrt{355}$ and $h(5) = \sqrt{495}$. Also, $h(3) = \sqrt{419}$ and $h(-2) = \sqrt{544}$. So the minimum value is $\sqrt{355}$ which occurs at $x = -5$ and the maximum value is $\sqrt{544}$ which occurs at $x = -2$.

8. (20 points) Compute the following definite integrals.

   (a) $\int_0^2 2xe^{-x^2}\,dx$

   Solution: $-e^{-x^2}]_0^2 = 1 - e^{-4} \approx 0.9816$.

   (b) $\int_0^5 (2x-1)\sqrt{x^2 - x + 5}\,dx$

   Solution: $2/3(x^2 - x + 5)^{3/2}]_0^5 = \frac{10}{3}(25 - \sqrt{5}) \approx 75.8798$.