May 6, 2019 **Name** ______________________________

The problems count as marked. The total number of points available is xxx. Throughout this test, **show your work.** Please note that a few problems that appear here were removed from the final because it was too long. Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (10 points) The function f satisfies $f(2) = 3$. The line tangent to f at $(2, 3)$ can be represented as $y - 3 = 5(x - 2)$, What is $f'(2)$. Explain your answer with a complete sentence.

2. (15 points) Consider the cubic curve $f(x) = 2x^3 - 4.5x^2 - 27x + 2$.

   (a) Build the sign chart for $f'(x)$.

   (b) Using the sign chart for $f'(x)$, find the intervals over which $f(x)$ is increasing.

   (c) Find a point of inflection on the graph of f.

3. (36 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. Find the critical points for each function and the intervals over which the function is increasing. You must show your work.

   (a) Let $F(x) = (2x + 8)(4x - 6)$

   (b) $G(x) = \dfrac{x^2 - 3x + 15/2}{2x - 1}$

   (c) $K(x) = (x^2 - 4)^{18}$

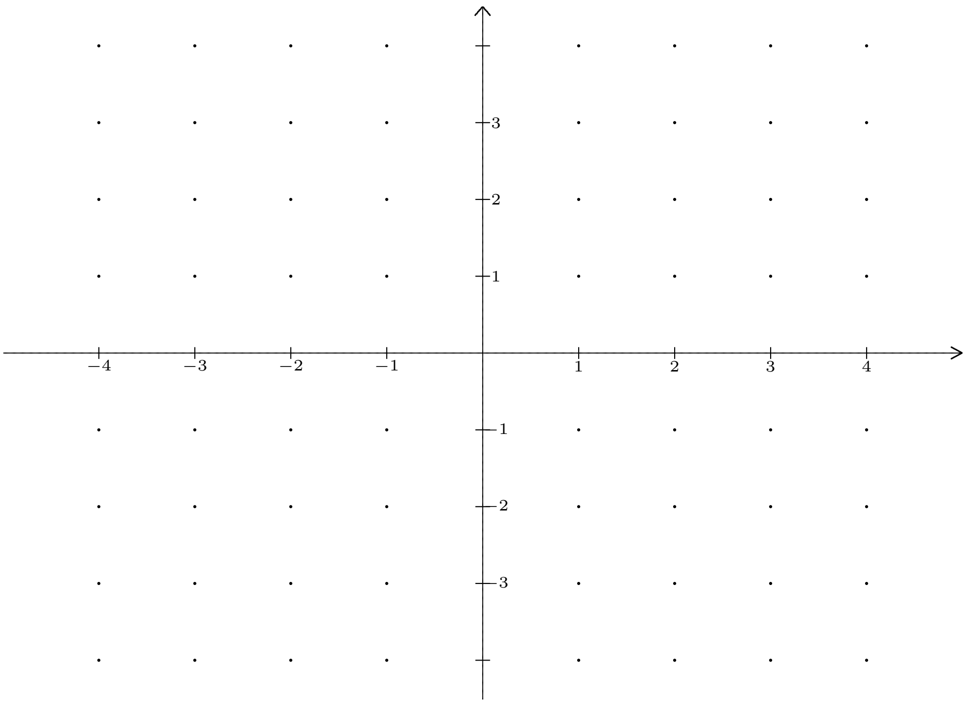
4. (30 points) Build a rational function $r(x) = \frac{p(x)}{q(x)}$ satisfying

   (a) r has vertical asymptotes at $x = \pm 3$

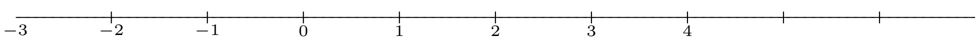
   (b) r has zeros at $x = 0$ and $x = 1$

   (c) r has the horizontal asymptote is $y = 2$.

   (d) Build the sign chart for your function. Finally, use all the information to sketch the graph of f.



(e) From the graph, you can speculate on the existence of critical points if there are any. Write a sentence about where you expect to find these critical points or why you think there are none. Estimate the sign chart for $r'(x)$



5. (20 points) Find a fourth degree polynomial function $p(x)$ which has the following properties:

   - p has relative minima at $x = -2$ and $x = 2$ and a relative maxima at $x = 0$,
   - $p(0) = 4$. Hint: how must the function $p'(x)$ factor in order to have the three critical points $x = 0, x = \pm 2$?

6. (10 points) The line tangent to the graph of $g(x)$ at the point $(4, 6)$ has a y-intercept of 9. What is $g'(4)$?

7. (10 points) Find all the points (x, y) on the graph of $h(x) = 2x^2 - 4x$ where the tangent line has a slope equal to 5.

8. (48 points) Find the following antiderivatives.

(a) $\int 2x - 5 \; dx$

(b) $\int 9x^2 - 4x - 2/x \; dx$

(c) $\int \frac{\ln(x)}{x} \; dx$

(d) $\int x(x-1)(x-2)^5 \; dx$

(e) $\int \frac{3x^3 + 2x^2 - x}{x} \; dx$

(f) $\int \frac{2x+3}{x^2+3x-3} \; dx$

(g) $\int 6x^5(x^6+3)^7 \; dx$

(h) $\int x^2 e^{x^3} \; dx$

(i) $\int \frac{2x+3}{x-1} \; dx$. Try substitution with $u = x - 1$.

(j) $\int (x+2)^4(x-2) \; dx$

9. (32 points) Find the following integrals.

   (a) $\int_0^1 \sqrt{5x+4}\ dx$

   (b) $\int_0^1 \frac{e^x}{e^x+1}\ dx$

   (c) $\int_1^6 |x-3|\ dx$

   (d) $\int_0^1 e^{\ln(x^3)}\ dx$

10. (10 points) Suppose you're given a function f to differentiate. You do so and you get $f'(x) = (x+3)(x+1)(x^2)(x-4)$. You conclude that f has four critical points, $x = -3, -1, 0$ and 4. Classify each of these as a) the location of a relative maximum, b) the location of a relative minimum, or c) an imposter.

11. (20 points) Consider the function $g(x) = \ln((x-3)(x^2+5x+4))$. Notice that $g(0) = \ln(-3 \cdot 4)$ is not defined because x must be positive in order to take ln of it. Find the domain of $g(x)$ and put your answer in interval notation. Then find the derivative of g and the critical points.

12. (20 points) Consider the five functions $f(x) = 3x + 4$, $g(x) = (3x - 2)^2$, $h(x) = \ln(x^5 + 1)$, $j(x) = e^{2x}$, and $k(x) = x^3 - x^2$.
    (a) Which of the functions is growing fastest at $x = 2$.
    (b) Put the functions in order from least fastest growth at $x = 2$ to the fastest growth at $x = 2$.
    (c) Put the functions in order from least to greatest by their growth over the interval $[1, 6]$.
13. (12 points) Let $H(x) = 2x - \ln(4x^2 + 12x + 10)$. Find all the critical points.
14. (20 points) Consider the function $f(x) = (2x - 4)e^{x^2}$.
    (a) Use the product rule to find $f'(x)$.
    (b) List the critical points of f.
    (c) Construct the sign chart for $f'(x)$.
    (d) Write in interval notation the interval(s) over which f is increasing.
15. (15 points) Find a number b such that $\displaystyle\int_b^{2b} x^3\,dx = 60$.

    You must show your work to get any credit on this problem. Guessing b is not enough.
16. (15 points) The region R is bounded by the vertical lines $x = 1$ and $x = e^2$ and by the graph of $f(x) = 1 + \frac{1}{x}$ and the x-axis. Find the area of R.

17. (30 points) Let $A = (6, 6)$, $B = (1, 4)$, $C = (1, 0)$ and $D = (6, 0)$ be the vertices of a quadrilateral in the plane.

   (a) Sketch the figure and use geometry to find the area of $ABCD$

   (b) Find an equation for the linear function (the line) that goes through the points A and B. Give this function the name f.

   (c) Use calculus to find the area of the region R defined as follows:

$$R = \{(x, y) : 1 \leq x \leq 6,\ 0 \leq y \leq f(x)\}$$

18. (10 points) Recall that a zero over zero problem is a limit problem of the form $\lim_{x\to a} \frac{f(x)}{g(x}$, where both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ are zero. Build a zero over zero problem

   (a) whose limit is 3.

   (b) whose limit is 0.

   (c) whose limit fails to exist.