

December 14, 2015

Name \_\_\_\_\_

The problems count as marked. The total number of points available is 280. Throughout this test, **show your work**. Using a calculator or a technique like L'hospital's Rule to circumvent ideas discussed in class will generally result in no credit. Throughout the test, express each measurement using the correct units.

1. (10 points) The line tangent to the graph of a function  $f$  at the point  $(2, 9)$  has slope  $-2$ . Find an equation for the tangent line in slope-intercept form.
  
  
  
  
  
  
  
  
  
  
2. (10 points) Find an equation in slope-intercept form for the line tangent to the graph of  $f(x) = x^{-2} - 3x$  at the point  $(2, f(2))$ ?
  
  
  
  
  
  
  
  
  
  
3. (10 points) Suppose  $f$  is a linear function. In other words, it can be written in the form  $f(x) = mx + b$  for some constants  $m$  and  $b$ . Suppose also that  $f(8) - f(2) = 12$ . What is  $m$ ?
  
  
  
  
  
  
  
  
  
  
4. (10 points) Find all values of  $x$  for which  $|x - 1| + |x - 5| + |x - 11| = 12$ .

5. (10 points) Find an equation for the line tangent to the graph of  $f(x) = \ln(3x + 1)$  at the point  $(0, 0)$ ?
6. (20 points) The marketing department of a large company has decided that the demand function for the new super-smartphones they manufacture is given by  $p = 12 - x^2$  where  $x$  is the number of (in thousands) produced each week and  $p$  is the price per phone in hundreds of dollars.
- (a) Build the weekly revenue function  $R(x)$  for the phone.
- (b) Find the marginal revenue.
- (c) How should the company price its phones to maximize the revenue?
- (d) Marketing has also established that the cost associated with the production of  $x$  phones is  $C(x) = 2.95x + 250$  again measured in hundreds of dollars. What is the average cost function?
- (e) What is the marginal average cost?

7. (40 points) Consider the function  $f$  below.

$$f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ x^2 - 1 & \text{if } -2 \leq x < 2 \\ 5 - x & \text{if } 2 < x \end{cases}$$

- (a) What is the domain of  $f$ ?
- (b) Discuss the continuity of  $f$  at the point  $x = -2$ .
- (c) Build the sign chart for  $f$ .
- (d) Find the derivative of  $f$ . Of course it will have to be piecewise defined, just like  $f$  is.
- (e) Find the intervals over which the function  $f$  is increasing.

8. (30 points) Let  $R$  denote the region defined by

$$R = \{(x, y) \mid 1 \leq x \leq e \text{ and } 0 \leq y \leq 1/x\}.$$

In other words,  $R$  is the region bounded by the lines  $x = 1$ ,  $x = e$  and  $y = 0$  (ie, the  $x$  axis), and the function  $y = 1/x$ .

- (a) Estimate the area of  $R$  by finding the area of a rectangular region inside  $R$ . The area of this rectangle is less than the area of  $R$ .
- (b) Estimate the area of  $R$  by finding the area of a rectangular region that encloses  $R$ . The area of this rectangle is greater than the area of  $R$ .
- (c) Compute the area of  $R$ .
- (d) How does the area of  $R$  change if we replace  $x = e$  with  $x = e^3$  in the definition of  $R$ ?

9. (50 points)

(a) Find  $\int_2^3 (x-2)(x^2+2x+4) dx$

(b) Find  $\int (x-2)^{10}(x+1) dx$

(c) Find  $\int (x-2)^2 dx$

(d) Find  $\int_{-2}^2 \sqrt{4-x^2} dx$

(e) Find  $\int x^3 - x^{-2} + x^{-1} dx$

(f) Evaluate  $\int_1^3 \frac{x^3-2x^2+x}{x} dx$

(g) Evaluate  $\int_0^8 \frac{d(x-5)^2}{dx} dx$

(h) Evaluate  $\int_0^4 \frac{3x^2}{x^3+5} dx$

10. (10 points) Let  $h(x) = \frac{\sqrt{(x-4)(x-2)(2x+9)}}{x^2-100}$ . Write the domain of  $h$  in interval notation.

11. (20 points) Consider the function  $F(x) = (x^2 + 1)^{\frac{3}{2}}$ .

(a) Find  $F'(x)$ .

(b) Use your work in (a) to compute  $\int \sqrt{x^2 + 1} \cdot 2x \, dx$ .

(c) Find  $\int_{-1}^1 \sqrt{x^2 + 1} \cdot 2x \, dx$ .

(d) Explain why you can get such an unusual answer as you did in part (c).

12. (15 points) It takes 14 years for a certain \$2000 continuously-compound investment to triple in value.

(a) How long does it take before its value is \$4000?

(b) How long does it take to reach a value of \$8000?

13. (15 points) It takes 7 years for a 60 units sample of *Reiterarium* to lose two-thirds of its radioactivity.

(a) What is the half-life of Reiterarium?

(b) How long does it take before the 60 units is reduced to 2 units?

14. (25 points) Let  $f(x) = e^{8-2x^2}$ .

(a) Compute  $f'(x)$

(b) Compute  $f''(x)$

(c) What is the slope of  $f$ 's steepest positive-sloped tangent line?

15. (15 points) Let  $f(x) = \ln[(x^2 + 1)(x^3 + x^2)(\frac{1}{x+2})]$ .

(a) Find the derivative of  $f(x)$ .

(b) What is  $f'(1)$ ?

(c) Find an equation for the line tangent to  $f$  at the point  $(1, f(1))$ . Leave your answer in terms of the  $\ln$  function.