June 26, 2001 Your name ____________________

The multiple choice problems count five points each. The total value of this test is 215 points.

1. Let $f(x) = \ln(x^4)$. What is $f'(e^2)$?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** $4e^{-2}$ **(E)** $4e^2$

2. Which of the following is closest to a solution to $2e^{x^2+1} = 2001$?

   (A) 1.74 **(B)** 2.43 **(C)** 6.91 **(D)** 10 **(E)** 31.59

3. For how many values of x is it true that

$$\log_3[(x^2-9)(x^2-4)+3] = 1?$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. Which of the following is closest to a solution of $\log_7(x+1) + \log_7(x+2) = \log_7 12$

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

From here on, it is important that you **show your work**.

5. (10 points) The doctor has told Mr. Tobigwaiste that he can expect a weight loss of 2% per week during the ten weeks of treatments he is about to begin. If his weight before the treatments begin is 244 pounds, what is his weight expected to be, to the nearest pound at the end of the ten weeks of treatments?

6. (10 points) A wolf population has decreased to 50 wolves but growth has begun at a rate of 1.2% per year. How many wolves, to the nearest whole number, can be expected in 10 years?

7. (10 points) Find an equation for the line tangent to the graph of $f(x) = e^{x^2+1}$ at the point $(1, e^2)$

8. (10 points) Solve each of the equations below for x in terms of the other letters.

   (a) $6 \cdot 2^{3x} = 18$

   (b) $\frac{1}{1+2^x} = \frac{2}{66}$.

9. (30 points) Suppose that the derivative of the function f is given by

$$f'(x) = x^2 - 6x + 5.$$

Note: you are given the *derivative* function! Answer the following questions about f.

(a) Find an interval over which f is increasing.

(b) Find the location of a relative maximum of f.

(c) Find the location of a relative minimum of f.

(d) Find an interval over which f is concave upwards.

(e) Suppose $f(1) = 3$. Find $f(2)$.

10. (20 points) Suppose the functions f and g are differentiable and their values at certain points are given in the table. The next four problems refer to these functions f and g. Notice that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. Note also that, for example, if $K(x) = f(x) - g(x)$, then $K'(x) = f'(x) - g'(x)$ and $K'(4) = f'(4) - g'(4) = 5 - 10 = -5$. Answer each of the questions below about functions that can be build using f and g.

x	$f(x)$	$f'(x)$
0	2	1
1	2	3
2	5	4
3	1	2
4	3	5
5	6	4
6	0	5
7	4	1

x	$g(x)$	$g'(x)$
0	5	5
1	7	3
2	4	6
3	2	6
4	6	10
5	3	3
6	1	2
7	0	1

(a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(4)$.

(b) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(2)$.

(c) The function H is defined by $H(x) = g(g(x))$. Use the chain rule to find $H'(2)$.

(d) Let $Q(x) = f(f(x) - g(x))$. Find $Q'(5)$.

(e) Find the derivative of the function f/g at the point $x = 4$.

11. (10 points) Compute each of the following derivatives.

    (a) $\frac{d}{dx}\sqrt{x^2+1}$

    (b) $\frac{d}{dx}\ln(x^4+1)$

12. (20 points) Compute the following antiderivatives.

    (a) $\int 6x^2 - 5x - 1 dx$

    (b) $\int 6x^{\frac{5}{2}} + x^{-\frac{1}{2}} dx$

    (c) $\int \frac{3x^2+2x-1}{x} dx$

    (d) $\int \frac{4x+1}{2x^2+x-3} dx$

13. (20 points) Compute the following integrals.

    (a) $\displaystyle\int_0^2 -3x^2 e^{-x^3}\,dx$

    (b) $\displaystyle\int_0^5 (2x+1)\sqrt{x^2+x+5}\,dx$

14. (10 points) Find the largest interval over which $f(x) = 4x^3 + 39x^2 - 42x$ is decreasing.

15. (10 points) Find a function $G(x)$ whose derivative is $3x^2 - 7x$ and whose value at $x = 4$ is 11.

16. (10 points) Find the area of the region bounded by $y = x^3 + 1$, the x-axis, and the lines $x = 0$ and $x = 4$.

17. (10 points) Find the area A of the region caught between the graphs of the functions
$$f(x) = -x^2 + 4x \text{ and } g(x) = -3x + 6.$$

18. (15 points) An apartment complex has 100 two-bedroom units for rent all at the same price. The monthly profit from renting x units is given by
$$P(x) = -10x^2 + 1760x - 50000$$
dollars. Find the number of units that should be rented out to maximize the profit. What is the maximum monthly profit realizable?