December 11, 2012 **Name** ______________________________

The total number of points available is 274. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (30 points) Domain Problems.
   For each function listed below, find the (implied) domain. Write your answer in interval notation.
   (a) $f(x) = \frac{x}{x^2+5x+6}$.
   (b) $g(x) = \sqrt{16 - \sqrt{x}}$.
   (c) $h(x) = \ln((x-3)(x-1)(x+3))$ Notice that $h(0) = \ln((-3)(-1)(3) = \ln 9$, so 0 belongs to the domain of h.
   (d) $k(x) = \sqrt{|x-1| - 3}$.

2. (30 points) Limit Problem
   (a) Find $\displaystyle\lim_{x\to -1} \frac{x^3 - x^2 + x + 3}{x^2 - 1}$.
   (b) Suppose $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$.
      i. Is it possible that $\lim_{x\to a} f(x) \cdot g(x) = 3$?
      ii. Is it possible that $\lim_{x\to a} f(x)/g(x) = 4$?
      iii. What are the possible outcomes of $\lim_{x\to a} \frac{f(x)}{g(x)}$? Can this limit fail to exist? Must the limit fail to exist? Write a sentence or two to show that you understand this question.

3. (20 points) A function $g(x)$ has been differentiated to get
   $$g'(x) = 2(x-5)^2 - 8.$$
   (a) Find the interval(s) over which $g(x)$ is increasing.
   (b) Find the interval(s) over which $g'(x)$ is increasing.
   (c) Find the interval(s) over which $g(x)$ is concave upwards.

4. (30 points) There is a cubic polynomial $p(x)$ with zeros at $x = -2, x = 1$, and $x = 2$.
   (a) Build one such function.
   (b) Build the sign chart for your function.

(c) Find an interval over which your function is increasing?

(d) Find the area of the region bounded by your function over the interval from $x = -2$ to $x = 1$.

5. (12 points) Given $f''(x) = 2x - 6$ and $f'(-2) = 6$ and $f(-2) = 1$. Find $f'(x)$ and $f(x)$.

6. (12 points) Let $f(x) = \dfrac{3}{x} - 2e^x$.

(a) Find an antiderivative of $f(x)$.

(b) Compute $\displaystyle\int_1^e f(x)dx$.

7. (42 points) Compute each of the following integrals

   (a) $\displaystyle\int_1^2 \frac{(4x-5)^2}{x}\,dx$

   (b) $\displaystyle\int_0^1 \frac{d}{dx}(x^3-2x^2+7)\,dx$

   (c) $\displaystyle\int_1^4 3x^2e^{x^3}\,dx$

   (d) $\displaystyle\int_2^3 \frac{x^3+2x^2-x}{x}\,dx$

   (e) $\displaystyle\int_1^3 \frac{2x+3}{x^2+3x-3}\,dx$

   (f) $\displaystyle\int_{-1}^1 6x^5(x^6+3)^7\,dx$

   (g) $\displaystyle\int_1^2 (x-1)^9x\,dx$

8. (15 points) Find the intervals over which $f(x)=x^2e^{2x}$ is increasing.

9. (12 points) Is there a value of b for which $\displaystyle\int_b^{2b} x^4+x^2\,dx = 128/15$? If so, find it.

10. (20 points) Use the substitution technique to find $\int (x-2)^4 \cdot x\,dx$. Then differentiate to check your answer.

11. (10 points) Find an interval where the function g defined by $g(x)=\ln(e^{x^2-4x})$ is increasing.

12. (10 points) Compute $\int \frac{d}{dx}\, xe^{x^2}\,dx$.

13. (15 points) Suppose x and y are positive real numbers satisfying $2xy=9$.

    (a) Find two pairs of numbers (x_1,y_1) and (x_2,y_2) satisfying the condition $2xy=9$. Compute the value of $2x+3y$ for each of these pairs.

    (b) What is the smallest possible value of $2x+3y$ such that $2xy=9$.

    (c) What is the smallest possible value of $3x+4y$ such that $2xy=9$.

14. (20 points) Use calculus to find the area of the trapezoid R bounded above by the graph of $f(x)=2x+1$, below by the x-axis, and on the sides by $x=1$ and $x=5$.