May 9, 2006 **Name** _____

The total number of points available is 250. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = xe^x$.

   (a) Find a value of x at which the line tangent to the graph of f is horizontal.

   (b) Find a value of x at which the line tangent to the graph of f has slope $2e$.

2. (10 points) Consider the function $f(x) = 24x^3 - 30x^2 + 14x - 2$. Find an antiderivative $F(x)$ of $f(x)$ that satisfies $F(1) = 2$.

3. (40 points) Suppose $u(x)$ is a function whose derivative is

$$u'(x) = (x^2 - 4)(x - 1)^2(x + 3)(3x + 17).$$

What this says is that u has already been differentiated and the function given is $u'(x)$. Recall that an important theorem tells you the intervals over which $u(x)$ is increasing based on $u'(x)$.

(a) Find the critical points of $u(x)$.

(b) Use the Test Interval Technique to find the intervals over which $u(x)$ is increasing.

4. (10 points) Consider the function $f(x) = x^{-1} - 2x^{-3}$. Let $F(x)$ be the antiderivative of $f(x)$ such that $F(1) = 0$. What is $F(x)$?

5. (10 points) Given $f''(x) = 2x - 6$ and $f'(-2) = 6$ and $f(-2) = 0$. Find $f'(x)$ and $f(x)$.

6. (10 points) Let $f(x) = \dfrac{7}{x} - 8e^x$. Find an antiderivative of $f(x)$.

7. (15 points) Is there a value of b for which $\int_b^{2b} x^4\,dx = 31/5$? If so, find it.

8. (15 points) What is the value of $\int_0^{\sqrt{15}} 2x\sqrt{x^2+3}\,dx$?

9. (15 points) Compound Interest.

   (a) Consider the equation $1000(1+0.02)^{4t} = 5000$. Find the value of t and interpret your answer in the language of compound interest.

   (b) Consider the equation $P(1+0.03)^{4\cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

   (c) Consider the equation $Pe^{10r} = 5P$. Solve for r and interpret your answer in the language of compound interest.

10. (20 points) Note that $g(x) = (x-1)(x-3)$ has two zeros in the interval $[0, 4]$.

    (a) Find the area of the region bounded by (i) the interval $[3, 4]$ on the x-axis, (ii) the line $x = 4$, and (iii) the graph of $g(x)$.

    (b) Compute $\int_0^4 g(x)\,dx$.

    (c) Find the area of the region caught between the graph of $g(x)$ and the x-axis over the interval from $x = 0$ to $x = 4$. Explain why this is different from the number found in part (b).

11. (30 points) A manufacture has been selling 1300 television sets a week at \$450 each. A market survey indicates that for each \$27 rebate offered to a buyer, the number of sets sold will increase by 270 per week. In other words, if they drop the price by \$27, they sell 270 more sets, etc.

    (a) Find the demand function $p(x)$, where x is the number of the television sets sold per week, and $p(x)$ is measured in dollars.

    (b) How large rebate should the company offer to a buyer, in order to maximize its revenue?

    (c) If the weekly cost function is $97500 + 150x$, how should it set the size of the rebate to maximize its profit?

12. (20 points) A rancher wants to fence in an area of 1000000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?

13. (20 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 165e^{-0.01t}$$

where $m(t)$ is measured in grams.

(a) Find the mass at time $t = 0$.

(b) How much of the mass remains after 15 years?

(c) What is the half-life of the material?

14. (20 points) Find the concavity of $f(x) = x^2e^{2x}$ over its domain.