Math 1120 **Calculus** **Final Exam**

August 11, 1999 Your name ____________________

1. Suppose the functions f and g are differentiable and their values at certain points are given in the table. The next four problems refer to these functions f and g. Notice that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. Note also that, for example, if $K(x) = f(x) - g(x)$, then $K'(x) = f'(x) - g'(x)$ and $K'(4) = f'(4) - g'(4) = 5 - 10 = -5$. Answer each of the questions below about functions that can be build using f and g.

x	$f(x)$	$f'(x)$
0	2	1
1	2	3
2	5	4
3	1	2
4	3	5
5	6	4
6	0	5
7	4	1

x	$g(x)$	$g'(x)$
0	5	5
1	7	3
2	4	6
3	2	6
4	6	10
5	3	3
6	1	2
7	0	1

(a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(3)$.

(b) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(1)$.

(c) The function H is defined by $H(x) = f(f(x))$. Use the chain rule to find $H'(2)$.

(d) Let $Q(x) = f(f(x) - g(x))$. Find $Q'(5)$.

(e) Find the derivative of the function f/g at the point $x = 4$.

2. Suppose that the derivative of the function f is given by

$$f'(x) = x^2 - 6x + 5.$$

Note: you are given the *derivative* function! Answer the following questions about f.

(a) Find an interval over which f is increasing.

(b) Find the location of a relative maximum of f.

(c) Find the location of a relative minimum of f.

(d) Find an interval over which f is concave upwards.

(e) Suppose $f(1) = 3$. Find $f(2)$.

3. Compute each of the following derivatives.

   (a) $\frac{d}{dx}\sqrt{x^3+1}$

   (b) $\frac{d}{dx}\ln(x^3+1)$

   (c) Let $f(x) = e^{x^2+1} \cdot e^{2x}$. Find $f'(x)$.

   (d) $\dfrac{d}{dx}\dfrac{e^x}{x}$

4. Compute the following antiderivatives.

   (a) $\int 6x^3 - 5x - 1dx$

   (b) $\int 6x^{\frac{3}{2}} + x^{-\frac{1}{2}}dx$

   (c) $\int \frac{3x^3 + 2x - 1}{x}dx$

   (d) $\int \frac{2x + 1}{x^2 + x - 3}dx$

5. Compute the following integrals.

   (a) $\int_0^2 2xe^{-x^2}\,dx$

   (b) $\int_0^5 (2x-1)\sqrt{x^2-x+5}\,dx$

6. Find the largest interval over which $f(x) = 4x^3 + 39x^2 - 42x$ is decreasing.

7. Find a function $G(x)$ whose derivative is $3x^2 - 7$ and whose value at $x = 4$ is 9.

8. Find the area of the region bounded by $y = x^{3/2}$, the x-axis, and the lines $x = 0$ and $x = 4$.

9. Find the area of the region caught between the graphs of the functions

$$f(x) = -x^2 + 4x \text{ and } g(x) = -2x + 5.$$

10. An apartment complex has 100 two-bedroom units for rent all at the same price. The monthly profit from renting x units is given by

$$P(x) = -10x^2 + 1760x - 50000$$

dollars. Find the number of units that should be rented out to maximize the profit. What is the maximum monthly profit realizable?