Name _____

We define three functions, f, g and h as follows: $f(x) = x^2 - x$, $g(x) = x + \frac{1}{x}$, and $h(x) = \sqrt{x+2}$. Notice that the derivatives of these functions are pretty straightforward: $f'(x) = 2x - 1; g'(x) = 1 - x^{-2}$; and $h'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}}$. Now the three functions f, g, h can be composed in six different ways. One of these is $F(x) = f \circ g \circ h(x)$. Let G, H, J, K, and L be the names of these functions. Find symbolic representations of each of these functions and their derivatives.

For example, $F(x) = (\sqrt{x+2} + 1/\sqrt{x+2})^2 - (\sqrt{x+2} + 1/\sqrt{x+2})$ and

$$F'(x) = 2\left(\sqrt{x+2} + 1/\sqrt{x+2}\right)\left(\frac{1}{2}(x+2)^{-1/2} - \frac{1}{2}(x+2)^{-3/2}\right) - \left(\frac{1}{2}(x+2)^{-1/2} - \frac{1}{2}(x+2)^{-3/2}\right).$$

Alternatively, you can write

$$\frac{d}{dx} f \circ g \circ h(x) = f'(g \circ h) \cdot \frac{d}{dx} g \circ h(x) = f'(g \circ h) \cdot g'(h(x)) \cdot h'(x),$$

and then fill in each function based on the calculations above.