1. Consider the game G_1 which starts with one pile of 10 counters. The rules allow a player to take $1, 3$, or 5 counters on each turn. The player who makes the last move wins. Denote this game by $N(10; 1, 3, 5)$. Construct the directed graph version of this game. It has 11 vertices labeled 0 through 10, and a directed edge from a vertex u to a vertex v if and only if $u - 1 = v$, $u - 3 = v$, or $u - 5 = v$. Show that you can partition the 11 vertices into two groups $\mathcal{S}$ and $\mathcal{U}$ so that each edge from a member of $\mathcal{S}$ goes to one in $\mathcal{U}$, and some edge from each member of $\mathcal{U}$ goes to one in $\mathcal{S}$, and the vertex labeled 0 belongs to $\mathcal{S}$.

2. Consider the game G_2 which starts with one pile of 20 counters. The rules allow a player to take $1, 2$, or 5 counters on each turn. Denote this game by $N(20; 1, 2, 5)$. Do the same analysis with this game.