

Consider the game of Bouton's nim with pile sizes 15, 20, 25, 30, 35.

1. Find the binary representation of each pile size.
2. Find the binary configuration of the game. That is, write these binary numbers in a column and compute their nim sum; Remember that to compute the nim sum, add the numbers with the understanding that  $1 + 1 = 0$ ,  $0 + 0 = 0$ ,  $1 + 0 = 0 + 1 = 1$ , and there is no carry from one column to another.
3. Notice that the binary configuration is not balanced since the nim sum of the pile sizes is not zero. Find a move which results in a balanced binary configuration. Is there just one such move or are there several?
4. Suppose you made a move which balances the configuration. Assume your opponent takes one counter from the same pile as the one from which you removed counters. What move do you make now?
5. Answer the same questions about each of the Bouton's Nim games listed below.
  - (a)  $N(16, 17, 18)$
  - (b)  $N(19, 27, 38)$
  - (c)  $N(16, 17, 18, 19, 27, 38)$