1. Some of the squares of an $m \times n$ grid of squares are covered by dominoes and the rest are empty. It so happens that none of the dominoes in place can be moved. They are blocked either by board boundary or another domino in each direction. Prove that at most one fourth of the squares are empty.

2. A checker board is painted white. At any time, we are allowed to choose any 3×1 rectangle and change the color of each of its squares (either from black to white or white to black). Is it possible to change all 64 squares to black?

3. Is it possible to tile the entire plane with $\times$ shaped pentominoes?

4. A 9×9 square is divided into 81 unit cells. Some cells are colored black so that the distance between the centers of any two black cells is at least 2.1. What is the largest number of cells that can be black? Explain why no more can be colored black.

5. All 81 squares of a 9×9 board are white. What is the fewest number which can be painted black so that every straight tetromino covers at least one black square?