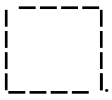
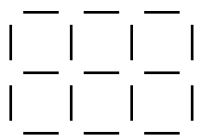
Julia Robinson Mathematics Festival Toothpick Problems

This set of problems involves the arrangement and rearrangement of toothpicks. Each toothpick can be considered a unit segment. Here are a couple of sample problems. The arrangement  is built from 16 toothpicks. Rearrange 10 of these to build a single square. Here's a solution.  .

Here's another example. The five toothpicks are used build a *gnu*: 
Now its pretty clear that we can't move both the neck and the head, so we should try the front or rear legs. The front legs don't work, but look at what we get if we move the rear legs. 
Have fun with the problems.

1. Notice that we can build a 2×3 grid of squares with 17 toothpicks.

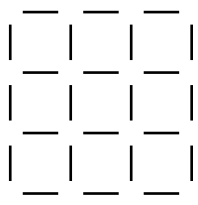


   How many toothpicks are required to build a 5×7 grid of squares.

2. How many squares of all sizes are outlined by the toothpicks in the 5×7 grid of squares?

3. In the same way as above, the 2×3 grid has 8 subsquares. Remove exactly four toothpicks to produce a figure with exactly 3 subsquares.

4. Next consider the 3×3 grid shown below.



(a) Can you remove exactly 8 toothpicks to produce a figure with exactly 2 squares?

(b) Can you remove exactly 8 toothpicks to produce a figure with exactly 3 squares?

(c) Can you remove exactly 8 toothpicks to produce a figure with exactly 4 *congruent* squares?

(d) Remove eight toothpicks leaving five squares.

(e) Move 12 toothpicks and make two congruent squares.

(f) Remove four toothpicks leaving one large and four small squares.

(g) Remove four toothpicks leaving 5 unit squares.

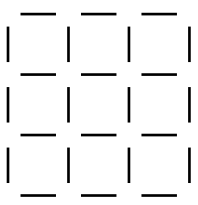
(h) Can you remove exactly 4 toothpicks to produce a figure with exactly 6 squares?

(i) Remove six toothpicks leaving three squares.

(j) Remove four toothpicks leaving nine squares.

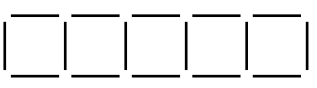
(k) Remove six toothpicks leaving two squares and two congruent hexagons.

5. Again consider the 3×3 grid shown.



How many rectangular regions can you find in this grid? Of course you should count squares as special cases of rectangles.

6. The next set of problems all deal with the 1×5 grid:



   (a) Rearrange exactly 8 toothpicks so that the new figure has two congruent squares.

   (b) Rearrange exactly 6 toothpicks so that the new figure has three squares.

   (c) Rearrange exactly 6 toothpicks so that the new figure has five squares.

7. In this problem we are building polygons with given areas using a given number of toothpicks.

   (a) Use exactly 14 toothpicks to build a polygon whose area is 12.

   (b) Use exactly 12 toothpicks to build a polygon whose area is 6.

   (c) Use exactly 16 toothpicks to build a polygon whose area is 12.

   (d) Using exactly 16 toothpicks, how many different areas can you capture with rectangles?

8. The house shown is built with 10 toothpicks. Rotate the house by rearranging just two toothpicks.